

User Guide

For

Stratagem EH5

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2190 Fortune Drive, San Jose, CA 95131 USA

Tel: (408) 954-0522 • Fax: (408) 954-0902 • geometrics.com

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1.0 Introduction

The Stratagem EH5 is a unique magnetotelluric (MT) system used to measure the electrical resistivity of the earth over depth ranges of a few meters to greater than one kilometer. This system uses naturally occurring electromagnetic signals to obtain a continuous electrical sounding of the earth beneath the measurement site. The natural signals are mostly the result of distant lightning activity in the atmosphere.

Investigators use the MT technique to measure the electrical impedance at the earth's surface by simultaneously collecting a series local electrical and magnetic field fluctuations. They make these field measurements over a period of several minutes, after which the data undergo Fourier transformation and then are stored as power spectra. The surface impedance calculated from these spectral values is a complex function of frequency where higher frequency data are influenced by shallow or nearby features, and lower frequency data are influenced by structures at greater depth and distance. An MT sounding provides an estimate of vertical resistivity beneath the receiver site and can indicate the geoelectric complexity at the sounding site. In areas where the resistivity distribution does not change rapidly from station to station, the resistivity sounding is a reasonable estimate of the geoelectric layering beneath the site.

Stratagem EH5 measurements may be effectively applied in ground water investigations, subsurface characterization at environmental study sites, mineral and geothermal exploration, and in civil engineering studies. Because this battery-powered system is compact and portable, it is productive even on rough terrain and may be operated successfully in hostile environments. The speed and portability of the Stratagem EH5 system allows for a flexible survey design. Surface impedance results are immediately displayed as a resistivity sounding and may also be processed in groups of soundings and viewed in section while on site. This flexibility enables the investigator to alter the survey design on site based on preliminary processing and examination of measurement results.

2.0 System Components

The following lists describe the components of a Stratagem EH5 system. The Standard receiver system is configured to acquire data in a frequency range from approximately 3 Hz to 70,000 Hz. The standard components and their interconnection are illustrated in figures 1 and 2.

2.1 Standard receiver components

- 4 ea. stainless steel electrode stakes
- 1 ea. system ground stake and ground cable

- 4 ea. BE-27 buffered electrodes with 27-meter telluric cables.
- 1 ea. EH5 receiver module
- 2 ea. G100K magnetic field coils
- 2 ea. C3/10 mag coil/receiver interconnect cables
- 1 ea. Operator's manual.
- 1 ea. Android or Windows tablet as system control and data console (optional)

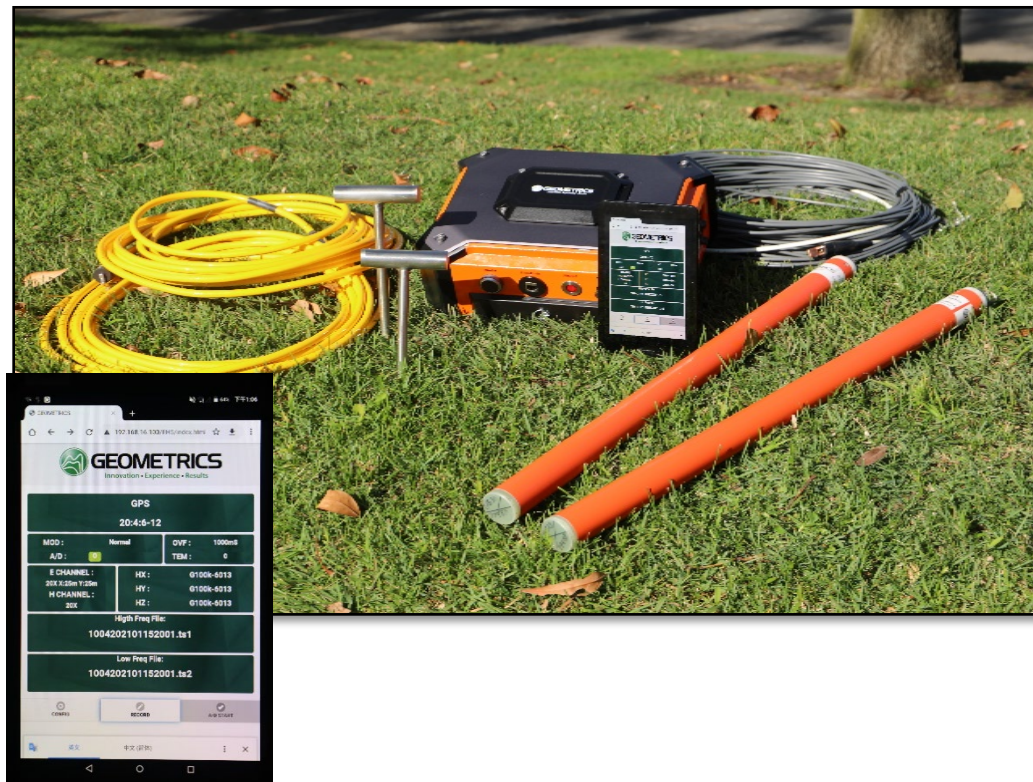


Figure 1. An illustration of the layout of the standard Stratagem system including EH5 receiver, stainless steel ground stakes, buffered electrode cables, G100K magnetic coils, mag coil cables. Inset photo shows a closeup of the Android controller.

3.0 Setting up

Setting up the survey configuration and data acquisition using MTPRO running on Windows PC or tablet.

3.1 Acquiring the EH5 WiFi connection:

Before acquisition can begin, the Android or Windows devices must detect the EH5 receiver WiFi. To do that do the following:

Turn on the EH5 receiver. When the receiver is first turned on the red LED will go into a rapid flash pattern indicating it is searching for the GPS satellite detection. Once it locks on to the GPS it will then go into a slower flash pattern.

Look for and select the **WiFi EH5****** where the **"*"** gives the name of the serial number of the EH5 receiver box. For example, WiFi "EH51011" would be the EH5 WiFi of receiver serial number 1011.

Open your browser on the Android or Windows device and enter <http://eh> and enter.

If it detects the EH5 WiFi it will display the time series screen below.

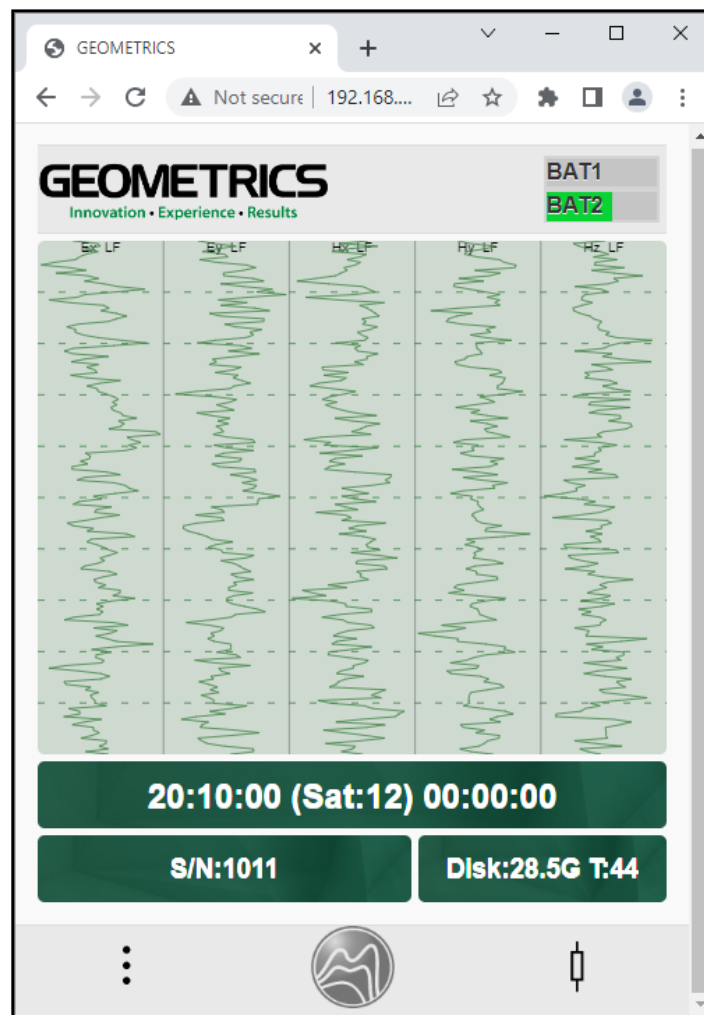

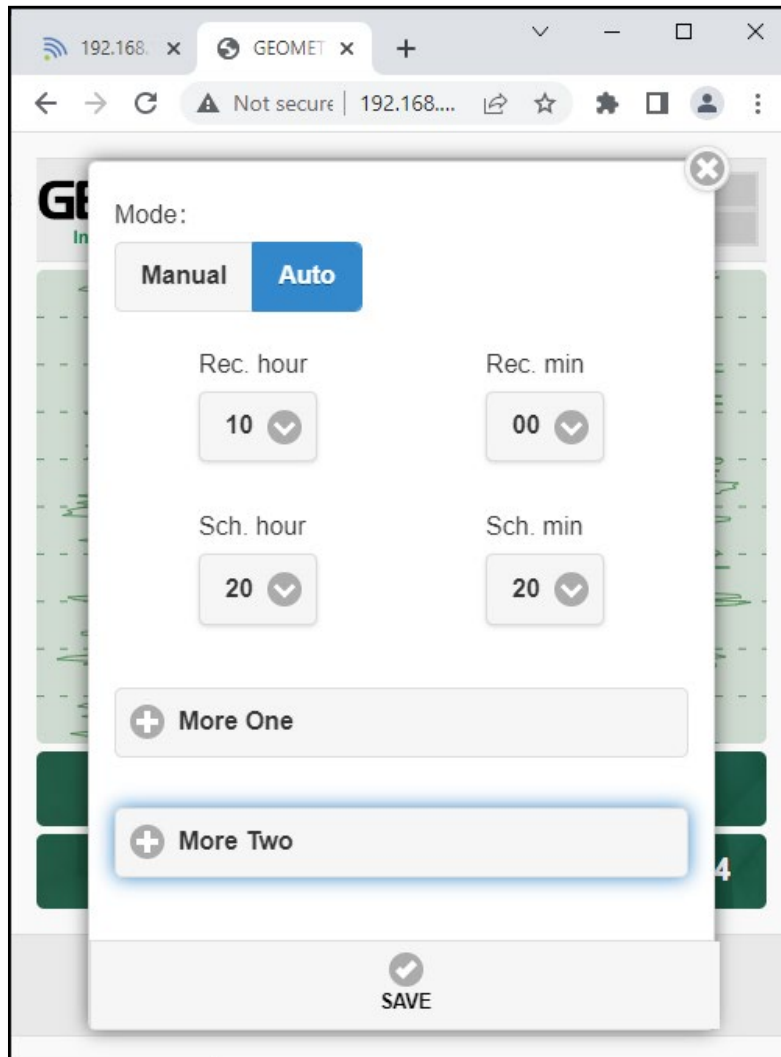


Fig. 2 EH5 acquisition screen showing time series.

3.2 Setting up the survey configuration

Click on the  icon to see the screen below.



Mode:

Manual **Auto**

Rec. hour: 10

Rec. min: 00

Sch. hour: 20

Sch. min: 20

+ More One

+ More Two

SAVE

Rec. hour used to set number of hours for the station acquisition time.

Rec. min used to set the number of minutes of acquisition.

When in the auto mode you set the time to start recording by setting the **Sch. hour** and **Sch. min** fields. The time used is UTC time from the GPS satellites. The current UTC time is displayed on the time series display menu. When in manual mode acquisition will start as soon as the start recording button is pressed, whether a scheduled hour and minute has been set or not.

Click on + **More One** to see the following menu:

More One

E GAIN:

01X

10X

20X

H GAIN:

01X

02X

05X

Power Line Frequency:

50Hz

60Hz

Ex Azimuth:

0.000

Declination:

13.000

Hx coil S/N:

G100k-1198


Hy coil S/N:

G100k-1201

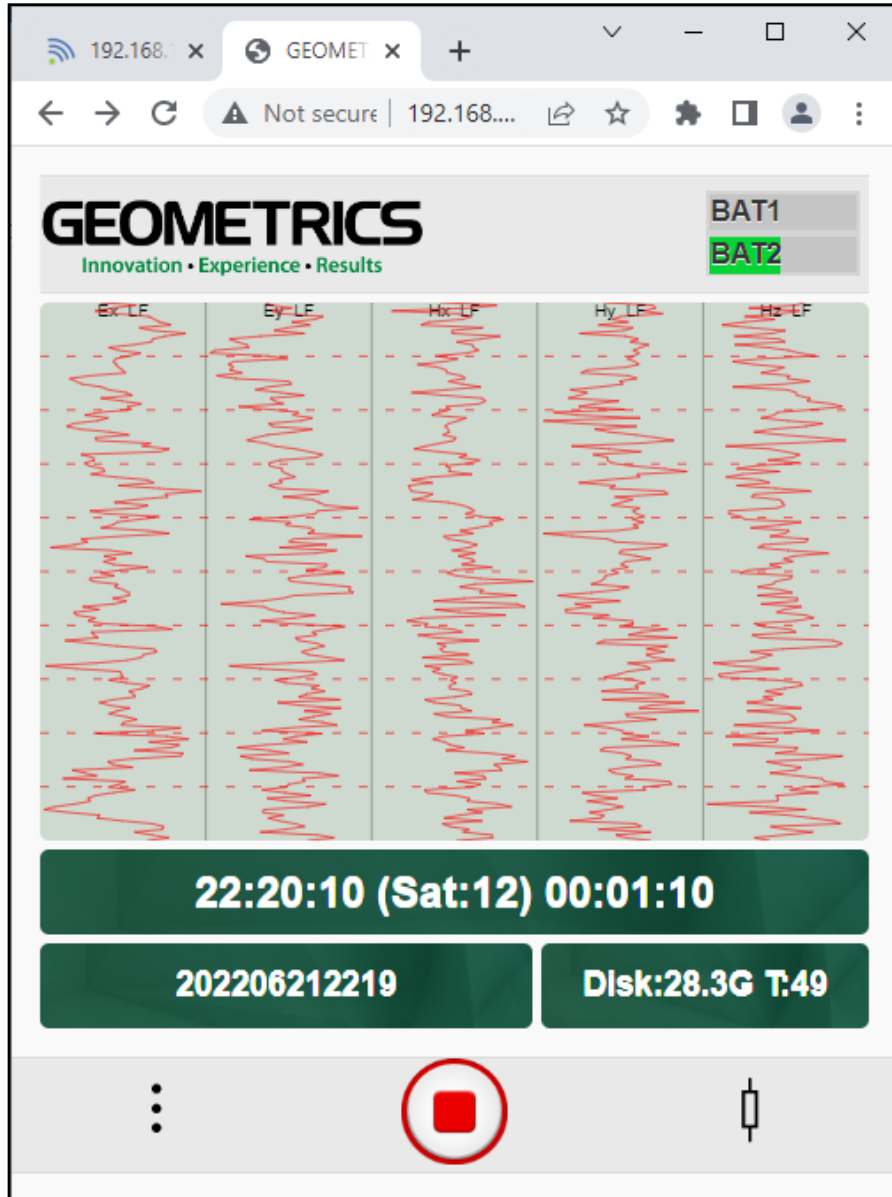
This menu allows you to set the survey parameters such as the line azimuth, coil calibration file names, dipole lengths, etc.

Click on + **More Two** to enter the operator name, site name, and comments


3.3 Doing acquisition

Click on the Geometrics  icon to see the acquisition menu below:

Before acquisition starts, the time series are display in **green**. During acquisiton, the time series and the Geometrics icon are displayed in **red** as shown below. There is a count down for up to 59 seconds to allow the data acquisiotn to synchronize to an exact minute.



The UTC time and acquisition time are displayed on the screen. Each acquisition will acquire both high and low frequency records. The labels at the top of the channel will indicate whether it is displaying high frequency (HF) or low frequency (LF) time series. The time series display is only a sample of the data. All the data is being stored in the EH5 receiver until it is manually transferred using EH5Pro to the Windows computer.

To stop acquisition **before** the acquisition time has expired, click on the red stop button, and click on OK. 

It will finish the record, stop recording, and the time series will turn **green**.

Note that the number in the field at the lower left corner of the acquisition screen is the name of the acquisition file. This is explained in more detail in the EH5Pro manual.

3.4 Transferring data from the EH5 Receiver to a Windows laptop for data processing

Use the program EH5Pro to transfer data files from the acquisition system to a Windows computer for processing.

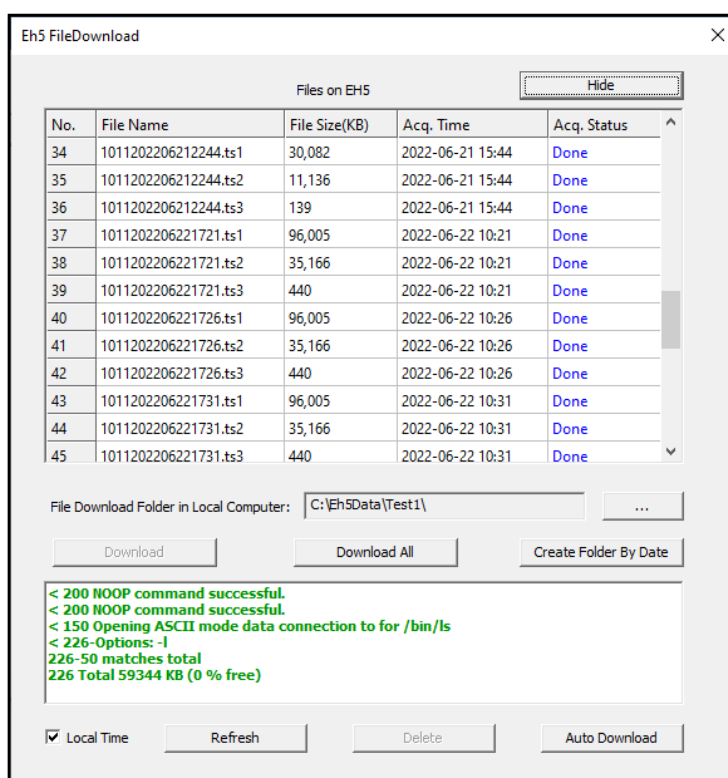
Do the following:

Turn on the EH5 receiver.

Confirm that your computer has detected and is on the EH5**** Wifi. This is described in section [3.1 Acquiring the EH5 WiFi connection](#) in this Operator's Manual.


Remember the log on code is 12345678.

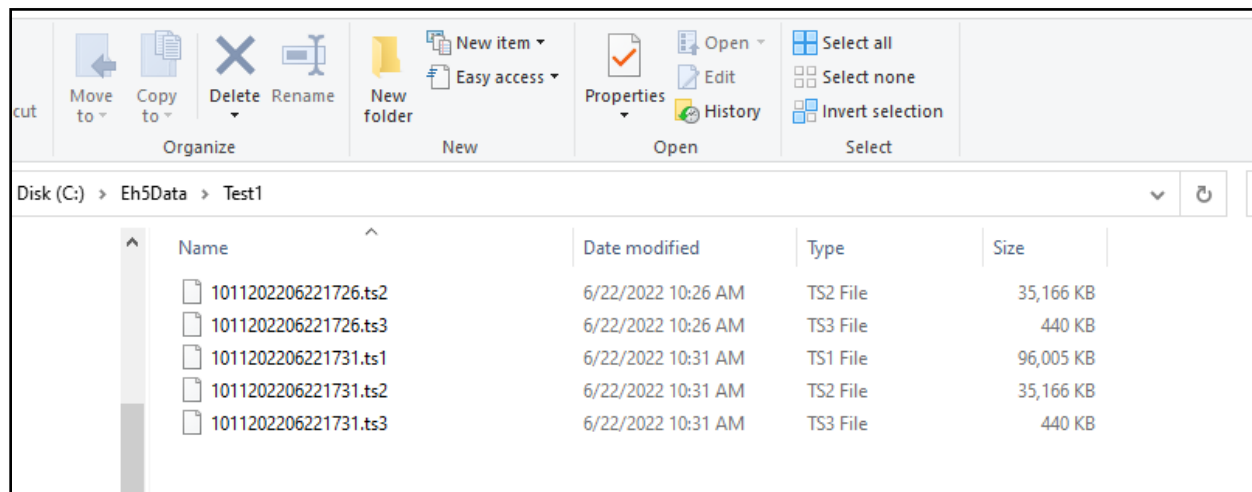
In the program EH5Pro go to “File\Download EH5 Files”. It will bring up the window below.



The word “**Done**” indicates that file has already been downloaded. Clicking on “**Create Folder By Date**” will create a new folder with the date as its name for downloading the selected files.

Use Windows Explorer if you want to create a folder with a different name and location.

Then click on the  button and navigate to the desired folder. For example: if the data is saved in "C:\EH5Data\Test1" you could find it in Windows Explorer as shown below:



The file naming conventions are explained in the EH5Pro Manual. As explained in the EH5Pro manual, data are saved by frequency range, as noted by their file extensions.

- **.ts1** files contain the high-frequency acquisition data.
- **.ts2** file contain the mid-frequency acquisition data.
- **.ts3** file contains the low-frequency acquisition data.

4.0 Parallel Noise Test

Parallel Noise Processing: The parallel noise test is a quality control tool for determining if the instrument and sensors are working properly. Parallel Noise Processing allows the operator to set up the two H coils parallel to one another, and the two electrodes parallel to one another but perpendicular to the two H coils. The parallel data is then used to display the noise level on the sensors and display parallel apparent resistivity and phase.

Parallel noise test field operation: In order to do the scalar parallel noise test in the field, data is acquired while the system is set up in parallel mode. Parallel mode means that both E-field measurements are parallel to one another and both H-field measurements are in parallel to one another but perpendicular to the E dipoles.

For example, if both E dipoles are oriented to 0 degrees, then both magnetic coils should be oriented to 90 degrees. Data is then acquired in the parallel mode. Details of processing and interpreting the parallel test data is found in the EH5Pro manual.

5.0 Field Operation

This section describes how to set up the Stratagem EH5 system and acquire soundings. We describe (1) receiver set-up and operation, (2) data collection and processing.

Prior to the physical deployment of the Stratagem hardware, you should have developed a survey plan. Creating a survey plan can be challenging, as your choices may affect the utility of the data set you produce. The survey plan is nothing more than the desired location of the various measurement sites and the frequency range recorded at these sites, but formulating an optimum survey plan will require that you consider limits on such things as field schedules, site access, acceptable horizontal resolution of the target structure, and the acceptable coverage of the target structure or the survey area. **If a survey area is new to you, it is helpful to setup the system and acquire a sounding at a site which you are certain will have high priority and then base a preliminary survey plan on your observations at this initial site.** Specific knowledge about the noise environment, geoelectric section, and the frequency range of interest will allow you to better estimate the recording time needed per sounding. This information will help in developing a realistic survey plan.

5.1 Receiver setup

All Stratagem cable connections are fitted with high-quality keyed, locking-ring connectors. These connectors are also capped. When cables are not connected, they should be capped to prevent moisture and debris from fouling the electrical connection. When cables are connected, the caps should be mated to keep their inner surfaces clean and dry.

It is simplest to begin a station setup by placing the EH5 receiver at the center of the measurement site and use this component as a reference point for installation of the other components. Because the EH5 measurements depend on the relative orientation of the sensors, it is also wise to select a reference direction for the survey. In the description that follows, this direction is referred to as 'X' direction. The 'Y' direction is 90° clockwise from 'X'. The complete layout is illustrated in Figure 1 and the recommended layout procedure follows.

5.2 Electrode installation

- A. If possible, select a site for the receiver, which is clear of surface impediments in the direction that the electrode cable will run.
- B. Install the system ground stake beside the receiver. Attach this stake (usually a spare electrode stake) to the post on the side of the AFE module with the system ground cable.
- C. With a compass, find the precise orientation ($\pm 2^\circ$), relative to the site's central point, of both ends of the two perpendicular lines for the electrode cable run (+ and - 'X' and 'Y').
- D. For each electrode, lay out the electrode cable to the desired distance and insert an electrode stake to about half its length in the earth at that point. If the earth is hard, it may be necessary to drive the electrode stake with your boot-heel or a hammer. DO NOT drive

an electrode stake while the buffered electrode is attached - these electrodes contain an active electronic circuit, which can be damaged by impact.

- E. After the electrode stake has been driven into the earth, water the stake to ensure good contact resistance between the stainless-steel stake and the ground.
- F. Return to the receiver to continue installing the remaining electric field sensors. On the way back, place the cable on the ground to minimize the effects of wind induced motion.
- G. Plug the electrode cables into the receiver; -'X' into X0, +'X' into X1, etc.

5.3 Coil installation

- A. Connect a coil cable (C1/10) to both the 'X' coil (G100K) and to the 'HX' connection on the receiver.
- B. Lay the coil several meters from the receiver on ground that is flat and level in the "X" direction. Use a shovel or hand-tool to excavate a suitable trench in which the coil can be leveled and aligned parallel to within +/- 2° of 'X'. Casting earth over the coil ensures that it will maintain its orientation, and this also reduces microphonic wind-induced noise and micro-seismic noise.
- C. Install the 'Y' coil following the same procedure used for the 'X' coil and locate it at least 2 meters from the 'X' coil. These coils are active electronic devices and, if placed too close together, they may interfere with one another. The coils also contain material with high magnetic susceptibility, which measurably distorts the local magnetic field. Because of this, the coils should be oriented using a technique which will keep the compass more than 0.5 meter from either coil.

6.0 Use of the Optional Hybrid-Source Transmitter

6.1 Caution:

The Geometrics hybrid-source transmitter must be used with care. In most cases the Stratagem EH5 AMT system can operation without the transmitter because it is sensitive enough to pick up the natural MT fields, even in the AMT dead zone between 1 kHz to 5 kHz. However, in some locations and under certain conditions, the transmitter can be useful. A better solution in most cases is to have a remote reference receiver. In the case of a remote reference there is no problem of the data being biased by the near field of the transmitter. There is also no need to move the transmitter several times as the measurements are moved along the profile line.

A basic assumption of MT processing is that the signal source must be in the far field. The minimum distance between the transmitter and the receiver for far field approximation is three times the skin depth. In fact, it is farther than that but the error in the data can be tolerated if the transmitter is at least 3 skin depths distance from the measurement. Skin depth is determined by both the frequency and resistivity with a good approximation of $502 \sqrt{\text{sq. rt. resistivity/frequency}}$. The lowest GEM3D HSAMT transmitter frequency is 1,200 Hz. In 10 Ohm-meter earth the far field approximation is 3 x skin depth. That would 137 meters. In 100 Ohm-meters it would be 435 meters. In 1,000 Ohm-meters it would be 1,506 meters.

The HSAMT transmitter is low power and runs on a standard marine or car 12 VDC battery. This limits the maximum distance the transmitter can be detected at approximately 500 meters. From the calculations above that means it can be used on a 10 Ohm-meters site with little concern. On a 100 Ohm-meter site it is near its limit of detectability of 500 meters. On a 1,000 Ohm-meter site it clearly cannot be used because at 500-meters distance it is well with in the far field.

7.0 Troubleshooting

The Stratagem EH5 system is designed to collect high frequency MT data in the range of 3 Hz to 70 kHz. If the system is not performing as you expect and the reason is not obvious, you will need to do some trouble shooting. This section contains a set of tables to quickly locate procedures most likely to resolve a problem with the system. We recommend that you read through this chapter completely before fielding the system for the first time.

7.1 Data Appraisal

Trouble must be observed before troubleshooting begins and, for the Stratagem system, this means reading and understanding the results seen in the various data displays. Described in the preceding section, these displays are the time-series display, the power spectra display, and the impedance results display.

7.1.1 Signal correlation

The naturally generated magnetotelluric field is the super-position of random signals at many frequencies and is expected to appear chaotic. Nevertheless, natural field events are usually correlated among all four channels. Also, the electric and magnetic fields are coupled for the sources we are interested in, so the time-series signature of a particular channel should appear like its complimentary orthogonal component. This means that the phase and amplitude of E_y should resemble H_x and the phase and amplitude of E_x should resemble H_y . You should be suspicious whenever one channel is frequently uncorrelated with the others or is constantly off scale.

Noise contamination or instrument malfunction of a more subtle nature may not be apparent in the time-series display but should be evident in the coherency data. **You should suspect an instrument problem if the coherency of one impedance mode is consistently low.** Also be suspicious when like channels (E_x - E_y or H_x - H_y) have consistently high spectral coherency. An extreme structural discontinuity could create these types of coherency relationships, but the more common culprits are cultural noise sources or component malfunctions.

7.1.2 Correlation of impedance components

A particular relationship is expected between the apparent resistivity curve and the impedance phase curve for valid sounding results. This relationship applies to either mode and is:

$$\phi = \frac{\pi}{4} \left(1 - \frac{\partial(\log \rho_a)}{\partial(\log T)} \right)$$

where ϕ is impedance phase, ρ_a is apparent resistivity, and T is period. It means that you should observe a dip in the phase response when you see an increase in the apparent resistivity as the period increases (frequency decreases). Conversely, an increase in the phase response should accompany a decrease in the apparent resistivity as period increases. Slight departures from this relationship are expected because of the normal low-level scatter in the data but when an entire portion of the sounding curve fails to obey it, the probable cause is that bias was introduced by cultural noise sources. As a rule, if the impedance phase is excessively high, the noise source is inductive. This type of noise can be observed by placing a transmitter too close to the receiver for the plane wave assumption to be valid. Similarly, noise signals that depress the phase below its expected value are due to grounded electrical sources. Soundings in remote areas with high contact resistance are apt to yield soundings with abnormally low impedance phase values for those portions of the sounding where natural field strength are low: the elevated electrode noise arising from the high contact resistance is the noise source. A similar effect is produced when spurious current is injected near the receiver site: The current in the noise source generates a magnetic field that is abnormally low relative to the voltage variations detected by the electrical field sensors.

7.2 Station placement

Is the system malfunctioning or is there is a problem with the measurement site? Measure sites should be placed well away from sources of electrical power generation and consumption and sites with mechanical activity. These sources include power lines of any size, electric fences, pipelines with cathodic protection, radio transmitters, metal windmills, and operating engines of all kinds. If you suspect that cultural noise is the source of a problem, its influence should disappear or be greatly reduced if you double the distance between it and the receiver site. Quiet receiver sites should avoid close proximity to large aggregations of metal such as stacked drill rod or irrigation pipe, railroads, metal sheds, and massive metal fences. Stock fencing is not a problem unless it is within several meters from the receiver coils.

Sensor vibration caused by the wind or flowing water will introduce noise into the sensors and, particularly when making measurements in the low frequency mode, will degrade the quality of the sounding. When making measurements in the high frequency mode, excessive vibration of the magnetic sensors may drive their output amplifiers into saturation. It is necessary to bury the magnetic sensors to reduce wind vibration to an acceptable level.

It is not always possible to avoid noisy sites, but it is important to differentiate between site noise and problems with the measurement equipment. For this reason, you should consider starting a survey with soundings at those sites that appear to be in an area relatively free of culturally generated noise. Later, if equipment problems are suspected, the first site can be reoccupied for a system check.

7.3 Contacting Geometrics

If you are unable to isolate or correct a problem with the Stratagem system, you should contact Geometrics and be prepared to provide specific information about the problem. You can reach our Customer Support department using support@geometrics.com. Communicating directly with Geometrics' technical staff is best when the data values appear to be incorrect. If this is impractical because of time zone differences or field schedules, try to send examples of the problem, either as

printout or data files. Also indicate which system you have and describe environmental conditions in the study area.

Appendix I Basic MT theory

Theory: Fundamental theory of MT and CSAMT operation: There are many books and other references on the theory of MT and CSAMT. For example, the SEG volume “Electromagnetic Methods in Applied Geophysics”, edited by Misac Nabighian. Also “Practical Magnetotellurics” by Simpson and Barh. And many others.

Electromagnetic waves

The electromagnetic waves of interest to the MT practitioner are described by Maxwell's equations. In differential form they are

$$\nabla \times \mathbf{E} = -i\omega\mu\mathbf{H} \text{ (Faraday's Law)} \quad (1)$$

$$\nabla \times \mathbf{H} = (\sigma + i\omega\varepsilon)\mathbf{E} \text{ (Ampere's Law)} \quad (2)$$

$$\nabla \cdot \mathbf{H} = 0 \quad (3)$$

$$\nabla \cdot \mathbf{E} = \rho/\varepsilon \text{ (Coulomb's Law)} \quad (4)$$

where \mathbf{E} is the electric field, \mathbf{H} is the magnetic field, σ is conductivity, ε is permittivity, and ρ is free electric charge. Here the notation \mathbf{E} and \mathbf{H} indicates vector harmonic time dependency.

$$e^{-i\omega t} = \cos(\omega t) - i \sin(\omega t)$$

so that the peak instantaneous vector value \mathcal{E} of \mathbf{E} is given by:

$$\mathcal{E} = \text{Re}(\mathbf{E}e^{i\omega t})$$

where ω is angular frequency ($2\pi f$), $i = \sqrt{-1}$, t is time and Re indicates the real part. A differential equation for \mathbf{E} is obtained by taking the curl of equation (1) and substituting equation (2) to get

$$\nabla \times \nabla \times \mathbf{E} = -i\omega\mu(\sigma + i\omega\varepsilon)\mathbf{E}$$

It is customary to let $k^2 = -i\omega\mu\sigma + \omega^2\mu\varepsilon$ and apply the vector identity:

$$\nabla \times \nabla \times \mathbf{A} = -\nabla^2 \mathbf{A} + \nabla(\nabla \cdot \mathbf{A})$$

to obtain:

$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0, \quad (5)$$

where we have also assumed that there is no free charge present ($\nabla \cdot \mathbf{E} = 0$). Equation (5) is the Helmholtz equation for \mathbf{E} and k is the propagation constant. The source of MT signals is in the atmosphere where the conductivity is near 0, and here the propagation constant is

$$k = \omega \sqrt{\mu_0 \epsilon_0}$$

In the earth

$$k \approx \sqrt{-i\omega\mu\sigma}$$

because, for earth materials, $\sigma \gg \omega\epsilon$ at the frequencies we are interested in.

Propagation of electromagnetic waves

A fundamental assumption when interpreting MT measurements is that the source fields impinge on the earth as uniform plane waves: the **E** and **H** fields are constant in planes perpendicular to the direction of propagation. For a plane wave propagating into a uniform earth, where the z direction is positive downward, we need only consider the field components E_x and H_y and can set the other components of the **E** and **H** fields to zero. With

$$\nabla^2 \times \mathbf{E} = \nabla^2 E_x \hat{x} + \nabla^2 E_y \hat{y} + \nabla^2 E_z \hat{z}$$

equation (5) becomes

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0 \quad (6)$$

because the plane wave does not vary in the x and y directions. This differential equation has a general solution

$$E_x = E_0^+ e^{-ikz} + E_0^- e^{ikz}$$

where the coefficients E_0^+ and E_0^- are constants which can be found by applying boundary conditions. Because of the requirement that a wave not gain amplitude in the direction of propagation, the coefficients E_0^+ and E_0^- represent the amplitudes of the electric field waves traveling in the +z direction (downward) and -z direction (upward) respectively. For a plane wave propagating in a uniform earth, E_x is 0 at an infinite depth because we are infinitely far from the plane wave source. This implies

that E_0^- represents the amplitude of a reflected wave, and in a uniform earth where there are no reflectors, its amplitude must also be zero. So

$$E_x = E_0^+ e^{-ikz} = E_0^+ e^{-iz\sqrt{-i\omega\mu\sigma}} = E_0^+ e^{-z(1+i)\sqrt{\omega\mu\sigma/2}}$$

or

$$E_x = E_0^+ e^{-z\sqrt{\omega\mu\sigma/2}} e^{-iz\sqrt{\omega\mu\sigma/2}} .$$

The term with the imaginary exponent represents the wave component of the electric field and the term with the real exponent defines the decay of the wave's amplitude with depth. It is convenient to specify distance in terms of the wavelength and, where EM waves penetrate conductors, it is customary to use one radian as the standard distance. This distance is called a skin depth. One wavelength is 2π radians so one skin depth is a bit less than one sixth of a wavelength. Wavelength is given by

$$\lambda = \frac{2\pi}{\kappa}$$

Where one skin depth is given by:

$$\delta = \frac{1}{\kappa}$$

and the wave number κ is:

$$\kappa = \sqrt{\frac{\omega\mu\sigma}{2}}$$

So that:

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{4\pi^2 10^{-7}}} \sqrt{\frac{\rho}{f}} \approx 503 \sqrt{\frac{\rho}{f}} \quad (\text{meters})$$

At the earth's surface ($z = 0$) and from equation 6, $E_x(0) = E_0^+$. At the skin depth δ , the amplitude of the down-going wave has decreased to:

$$E_x(\delta) = E_0^+ e^{-1}$$

which is $1/e$ or about 37 percent of its surface value. If a reflective layer existed at a depth δ , then the reflected, up-going wave would also be attenuated by 37 percent resulting in a total attenuation of about 86 percent relative to the surface value of the down going wave.

The presence of uniform plane-waves implies that the wave's source is distant: nearby sources are likely to generate waves with a spherical wave front that will not be uniform in the survey area. If sources are nearby, equations 1 and 2 are incomplete; they lack the appropriate source terms for a complete description of the fields and their interaction. Because it is difficult to specify sources that are beyond our control, it is best if they can be avoided. Both experimental results and numerical simulations indicate that at distances greater than 3 skin depths ($1/2$ wavelength) from an electromagnetic transmitter, the uniform, and plane portion of the waves are dominant and at 6 or 7 skin depths (1 wavelength) the waves are completely uniform and plane relative to the precision with which we can measure them.

Impedance

In the air, above a uniform earth, we expect to find an electromagnetic field composed of electromagnetic waves propagating downward and reflected waves propagating upward. This situation is described for an electric field component by

$$E_x^0 = E_0^+ e^{-ik_0 z} + E_0^- e^{-ik_0 z} \quad (7)$$

The coefficient subscripts '0' indicate that this expression applies to layer 0, (the air) and the superscripts indicate the direction of travel (positive down). In a uniform earth (layer 1) there are no reflectors so:

$$E_x^1 = E_1^+ e^{-ik_1 z} \quad (8)$$

The propagation constants in these expressions are:

$$k_0 = \omega \sqrt{\mu_0 \epsilon_0} \quad \text{and} \quad k_1 = e^{i\pi/4} \sqrt{\frac{\omega \mu}{\rho}}.$$

A solution for the unknown constants in equations 7 and 8 is obtained by enforcing the conditions that E_x and H_y must be continuous at the earth-air boundary. Applying Faraday's law to 7 and 8 gives the magnetic fields

$$H_y^0 = \frac{E_0^+}{\eta_0} e^{-ik_0 z} + \frac{E_0^-}{\eta_0} e^{-ik_0 z} \quad \text{and} \quad H_y^1 = \frac{E_1^+}{\eta_1} e^{-ik_1 z}$$

where η_0 and η_1 are the intrinsic impedance of the air and the earth given by:

$$\eta_0 = \frac{i\omega\mu_0}{ik_0} = \sqrt{\frac{\mu_0}{\epsilon_0}} \quad \text{and} \quad \eta_1 = \frac{i\omega\mu_0}{ik_1} = \sqrt{\omega\mu_0\rho} e^{i\pi/4}$$

Equating E and H fields at $z = 0$ yields

$$E_1^+ = E_0^+ + E_0^- \quad \text{and} \quad \frac{E_1^+}{\eta_1} = \frac{E_0^+}{\eta_0} - \frac{E_0^-}{\eta_0}.$$

The solution of these equations yields the amplitude of the transmitted wave and the reflected wave:

$$E_1^+ = \frac{2\eta_1}{\eta_0 + \eta_1} E_0^+ \quad \text{and} \quad E_0^- = \frac{\eta_1 - \eta_0}{\eta_0 + \eta_1} E_0^+.$$

These equations lead to expressions for the electric and magnetic field in the air in terms of impedance of the air and earth given by:

$$E_x^0 = E_0^+ \left[e^{-ik_0 z} + \frac{\eta_1 - \eta_0}{\eta_1 + \eta_0} e^{-ik_0 z} \right] \quad \text{and} \quad H_y^0 = \frac{E_0^+}{\eta_0} \left[e^{-ik_0 z} - \frac{\eta_1 - \eta_0}{\eta_1 + \eta_0} e^{-ik_0 z} \right].$$

Similarly, in the earth we have:

$$E_y^1 = E_0^+ \frac{2\eta_1}{\eta_1 + \eta_0} e^{-ik_1 z} \quad \text{and} \quad H_v^1 = E_0^+ \frac{2}{\eta_1 + \eta_0} e^{-ik_1 z}.$$

At the surface of the earth, these two sets of expressions are equal and the ratio of E_x/H_y is called the surface impedance \mathbf{Z} . For the case of a homogeneous earth, $\mathbf{Z} = \boldsymbol{\eta}$. This is the basis for defining the apparent resistivity ρ_a : it is the resistivity of a homogeneous earth, which would yield the same surface impedance as measured over an inhomogeneous earth at a particular location and frequency. Because

$$\eta_1 = \frac{E_x}{H_y} = e^{i\pi/4} \sqrt{\omega \mu_0 \rho}$$

we can write:

$$\rho_a = -\frac{i}{\omega \mu_0} \left(\frac{E_x}{H_y} \right)^2$$

In general, apparent resistivity is complex (e.g. the IP phenomena), but we restrict our attention to the real component which is defined as,

$$\rho_a = -\frac{i}{\omega \mu_0} \frac{|E_x|^2}{|H_y|^2}$$

Impedance estimation

Modern magnetotelluric systems are designed to record variation of both the electric and magnetic fields in two orthogonal directions and use these records to calculate the surface impedance at a measurement site. The surface impedance \mathbf{Z} is complex, frequency dependent, and, due to the presence of noise and earth structures, is also a tensor:

$$\begin{aligned} E_x &= Z_{xx} H_x + Z_{xy} H_y \\ \overline{E}(\omega) &= \overline{\mathbf{Z}}(\omega) \cdot \overline{H}(\omega) \quad \text{or} \quad E_y = Z_{yx} H_x + Z_{yy} H_y \end{aligned}$$

It can be helpful to think of the surface impedance tensor as a two input, two output linear system where the inputs are the magnetic field components, and the outputs are the electrical field components. This formulation of surface impedance is preferred over the scalar formulation because, when the source fields are nearly plane waves, the impedance elements \mathbf{Z}_{ij} are time invariant. The scalar surface impedance

is easier to calculate but can vary as the direction of the source fields vary.

$$Z_{ij} = \frac{E_i}{H_j}$$

The tensor impedance can be calculated from several records (N) using the least-squares method where the difference between a measured electric field component is minimized relative to the predicted electric field component. For example, just considering **Ex**, **Hx** and **Hy**

$$\psi = \sum_{i=1}^N (E_{xi} - Z_{xx} H_{xi} - Z_{xy} H_{yi}) (E_{xi}^* - Z_{xx}^* H_{xi}^* - Z_{xy}^* H_{yi}^*)$$

and minimization with respect to **Zxx** and **Zxy** requires

$$\frac{\partial \psi}{\partial \text{re} Z_{xx}} = \frac{\partial \psi}{\partial \text{im} Z_{xx}} \quad \text{and} \quad \frac{\partial \psi}{\partial \text{re} Z_{xy}} = \frac{\partial \psi}{\partial \text{im} Z_{xy}}$$

which yields

$$\langle E_x H_x^* \rangle = \langle H_x H_x^* \rangle Z_{xx} + \langle H_y H_x^* \rangle Z_{xy} \quad \text{and} \quad \langle E_x H_y^* \rangle = \langle H_x H_y^* \rangle Z_{xx} + \langle H_y H_y^* \rangle Z_{xy}$$

where, for, example

$$\langle E_x H_x^* \rangle = \frac{1}{N} \sum_{i=1}^N E_{xi} H_{xi}^*$$

is an average cross power density spectrum and **Ex** is the discrete Fourier transform of the measured field **Ex**. The (*) symbol indicates complex conjugation. The two cross power expressions above combine to yield

$$Z_{xx} = \frac{\langle E_x H_y^* \rangle \langle H_y H_x^* \rangle - \langle E_x H_x^* \rangle \langle H_y H_y^* \rangle}{\langle H_x H_y^* \rangle \langle H_y H_x^* \rangle - \langle H_x H_x^* \rangle \langle H_y H_y^* \rangle} \quad \text{and} \quad Z_{xy} = \frac{\langle E_x H_y^* \rangle \langle H_x H_x^* \rangle - \langle E_x H_x^* \rangle \langle H_x H_y^* \rangle}{\langle H_y H_y^* \rangle \langle H_x H_x^* \rangle - \langle H_y H_x^* \rangle \langle H_x H_y^* \rangle}$$

and expressions for **Zyx** and **Zyy** are obtained in a similar way.

The surface impedance is usually expressed as apparent resistivity and impedance phase and they are calculated from the surface impedance components as

$$\rho_{ij} = \frac{1}{\omega\mu_0} |Z_{ij}|^2 = \frac{.2}{f} |Z_{ij}|^2 \quad \text{and} \quad \phi_{ij} = \tan^{-1} \left(\frac{\text{Im}(Z_{ij})}{\text{Re}(Z_{ij})} \right)$$

whether they are based on scalar or tensor calculations.