Data Acquisition and Analysis of Active and Passive Surface Wave Methods

SAGEEP 2003 Short Course

OYO Corporation
Koichi Hayashi

Outline

- Introduction to Surface Waves
- Fourier Transform
- Phase Velocity and Dispersion Curves
- Inversion
- Active Method
- Passive Method
- Application to Engineering Problems

Introduction to Surface Waves

- Introduction to Surface Waves
- Fourier Transform
- Phase Velocity and Dispersion Curves
- Inversion
- Active Method
- Passive Method
- Application to Engineering Problems

Geophysical explorations using surface-waves

Classification	Method	Name	Application
Seismic	Refraction	Refraction method	*Tunnel*
		PS logging	*Dam*
		Cross-hole tomography	*Earth sciences*
	Reflection		
	Surface-wave	□	
		Micro-tremor array	
		Surface wave method	
		□	
Electro-magnetic	DC resistivity	2D DC resistivity method	
		Logging	
	Electro-magnetic		
	Ground penetrating radar		
Gravity			

Geophysical explorations using surface-waves

Exploration using surface waves

Micro-tremor array measurements

- From tens of meter to several kilometers
- 0.2Hz～10Hz
- Micro-tremors
- Long period seismometer

Surface wave methods

- 1m to 15m
- 5Hz to 50Hz
- Artificial vibration (sledge hammer, etc.)
- Geophones

Earth sciences

Investigation of deep ground

Understanding of the bearing stratum

Reclaimed ground

River bank

Embankment / Housing land development

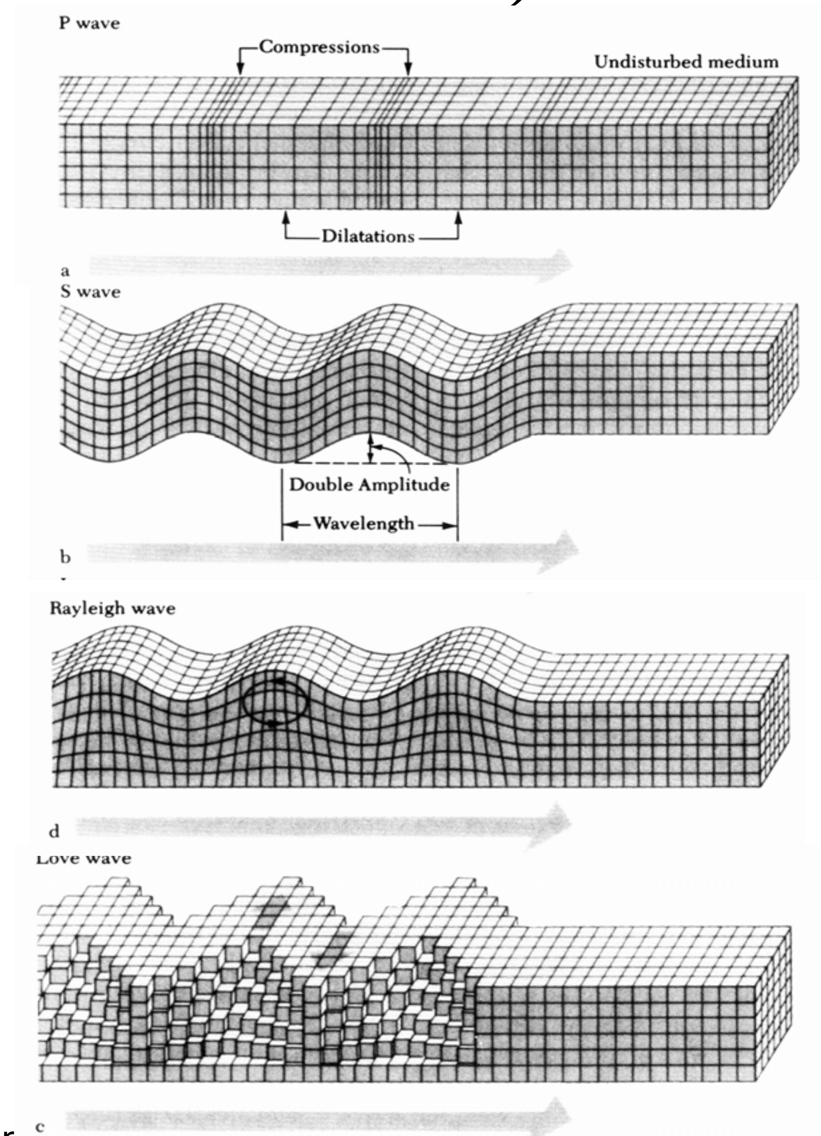
Fundamentals of surface-waves (Seismic waves)

- Body waves
  P-wave

  S-wave

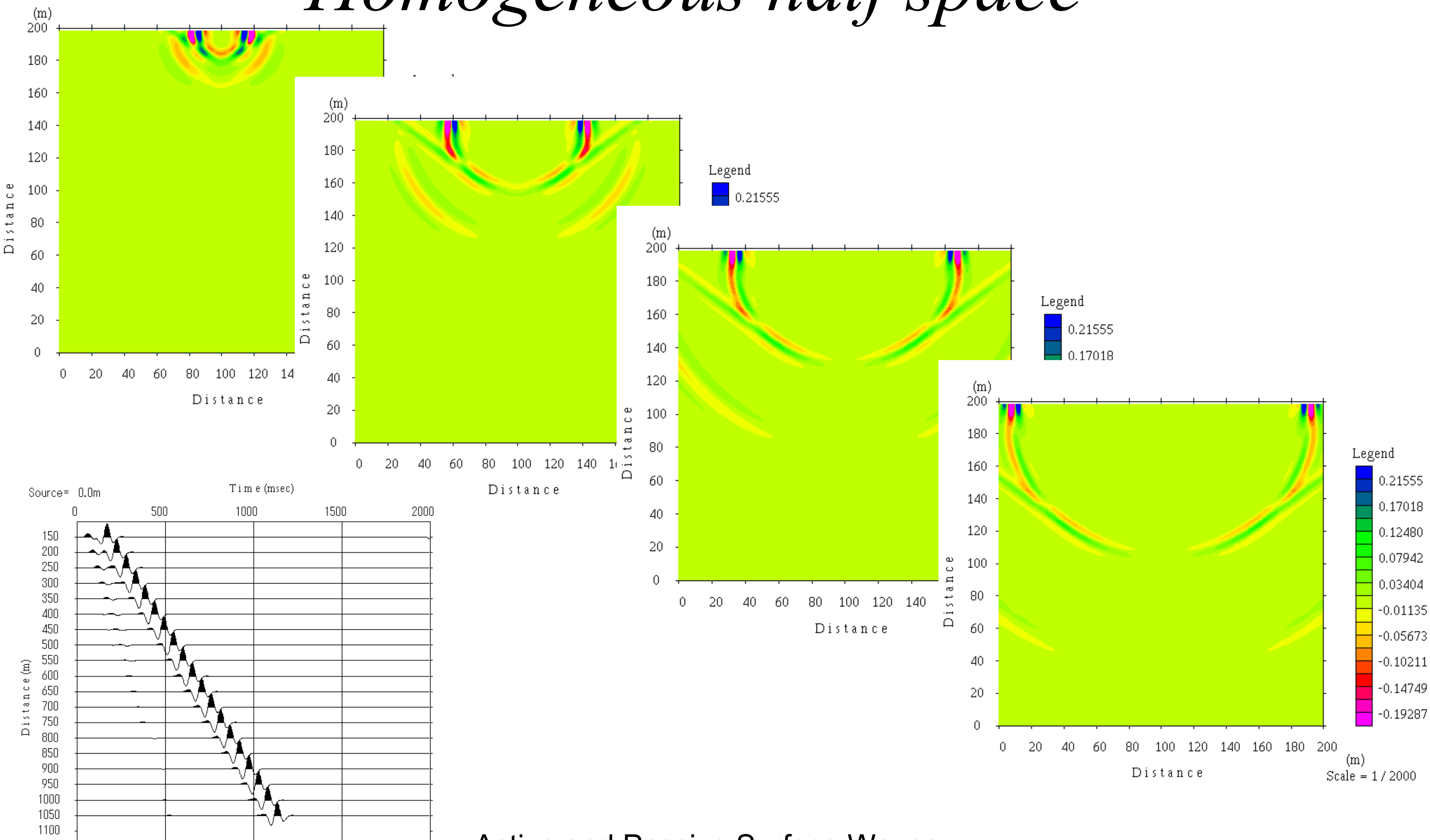
- Surface-waves
  Rayleigh-wave

  Love-wave

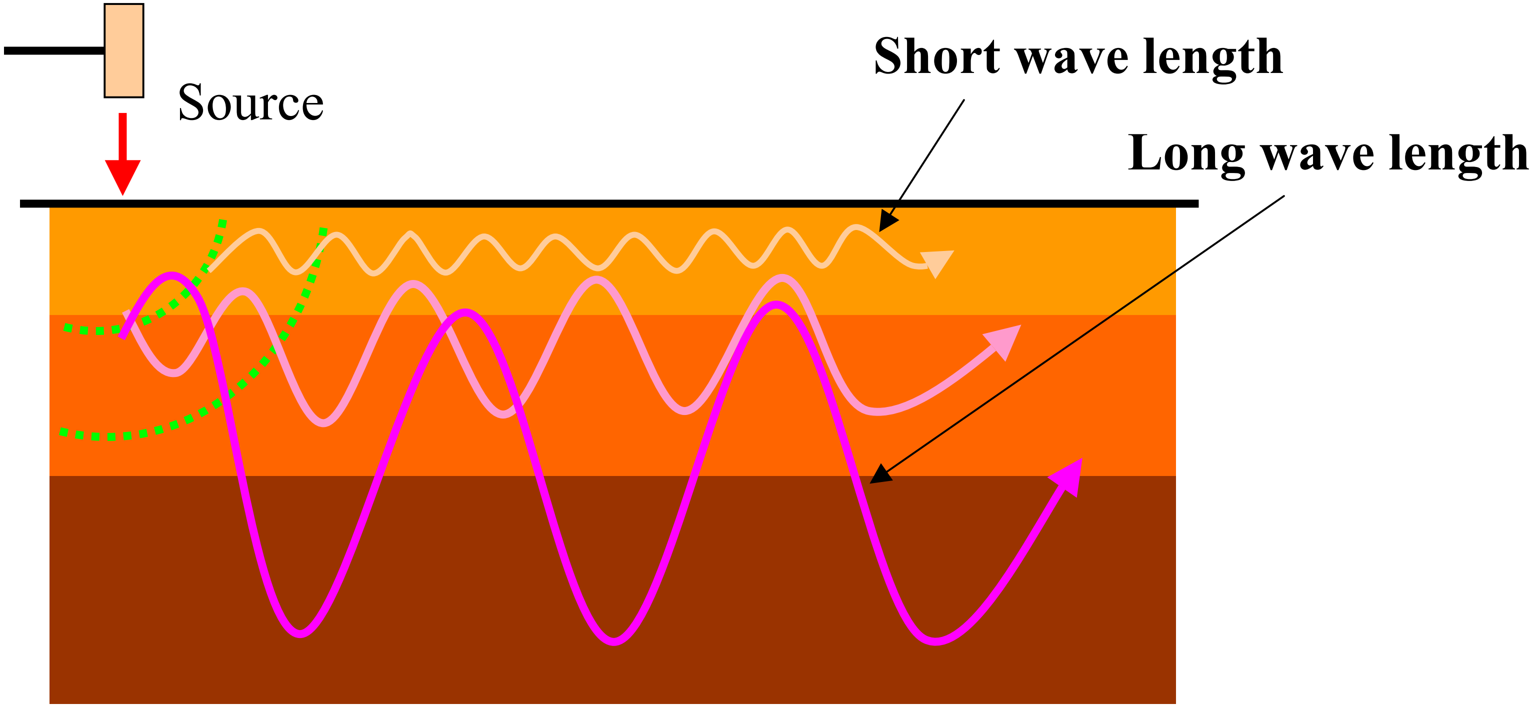


Propagation of surface-waves (Rayleigh -wave)

Homogeneous half space



Surface-wave dispersion
Heterogeneous medium

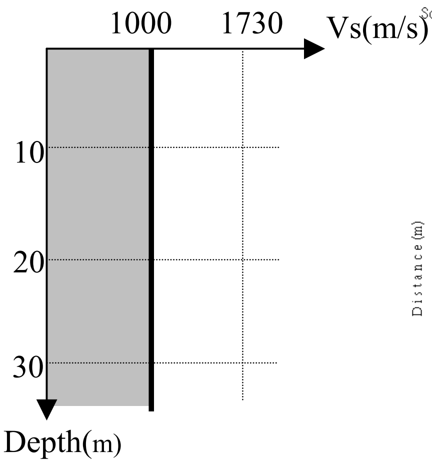


Phase velocity differs in the frequency.

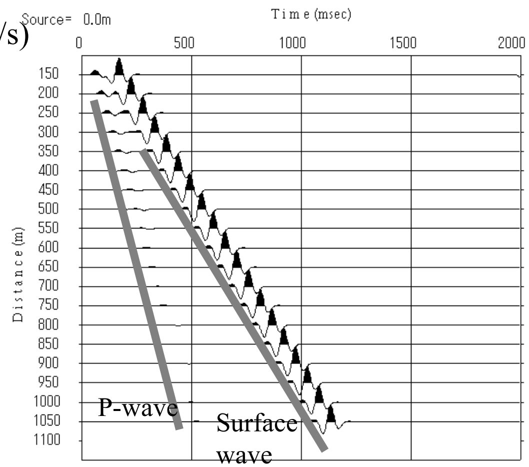
□Dispersion

Surface-wave dispersion

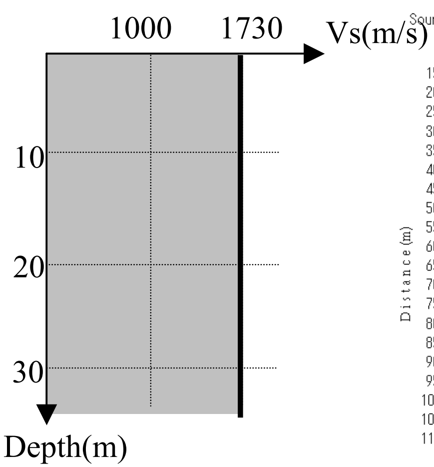
Homogeneous model



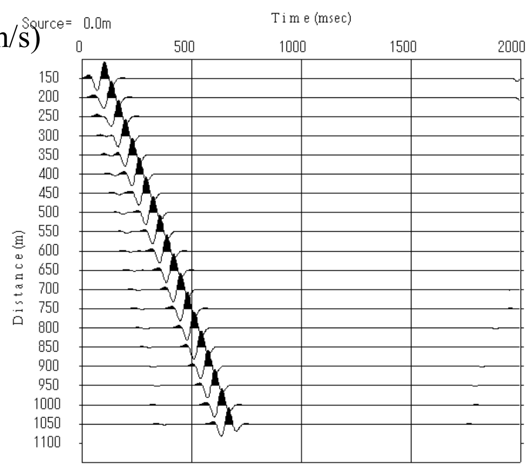
S-wave velocity model



Theoretical waveform

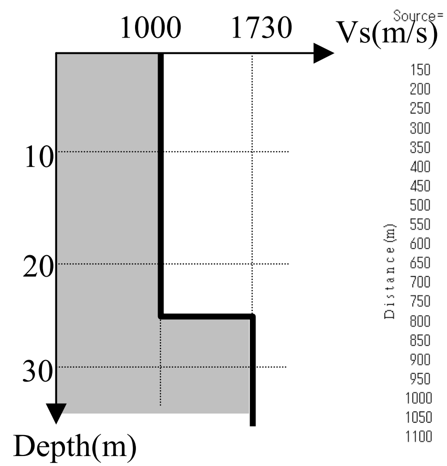


S-wave velocity model

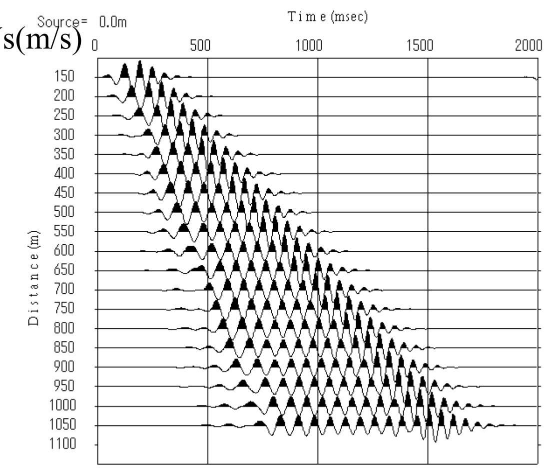


Theoretical waveform

Two-layer model



S-wave velocity model



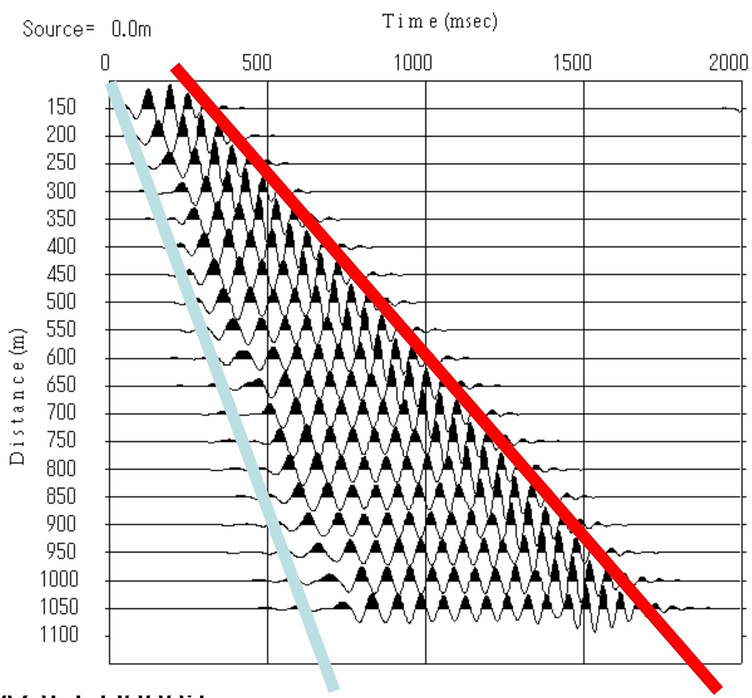
Theoretical waveform

Surface-wave dispersion

Q：What is “dispersion”?

A：Phase velocity differs in the frequency

　→The velocity of each frequency is called Phase velocity.



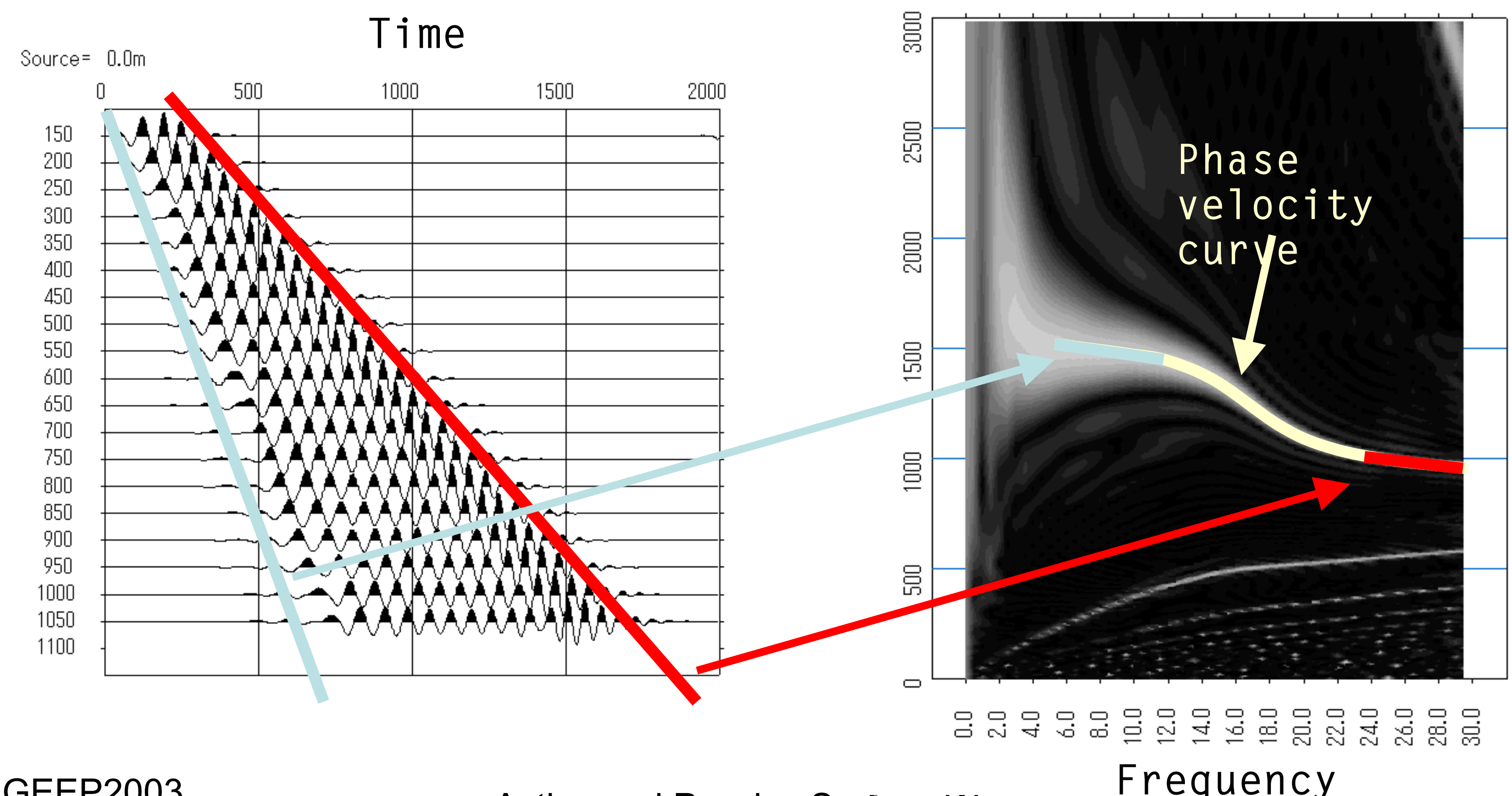
The wave of long period (low frequency) transmits fast, and the wave of short period (high frequency) does slowly.

→Dispersion

Surface-wave dispersion

Q: What is "Phase velocity curve (Dispersion curve)"?

A: It is the graph of phase velocity representing with frequency and velocity.



Characteristics of surface-wave (Rayleigh-wave) survey

- Phase velocity of surface-wave is sensitive to the S-wave velocity.
- Phase velocity of surface-wave us 0.9 to 0.95 times that of S-wave.
- Difference of wave length causes the difference of survey depth.
- Increasing source power efficiency
  Surface-waves : 67%, S-wave: 26%, P-wave: 7%
- Easy to survey
- Possible to survey low velocity layer underneath high velocity one.

Fourier Transform

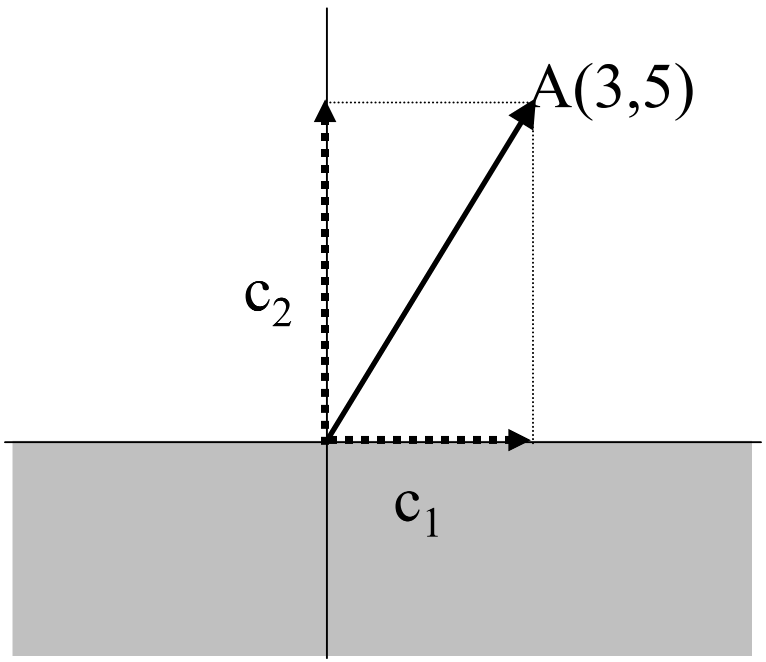
- Introduction to Surface Waves
- Fourier Transform
- Phase Velocity and Dispersion Curves
- Inversion
- Active Method
- Passive Method
- Application to Engineering Problems

Fourier transform is

1. Orthogonal
   The inner product of any two different harmonics is zero
2. Complete
   No function can be orthogonal to all harmonics
3. Convenient
   Can be calculate efficiency

Vector decomposition

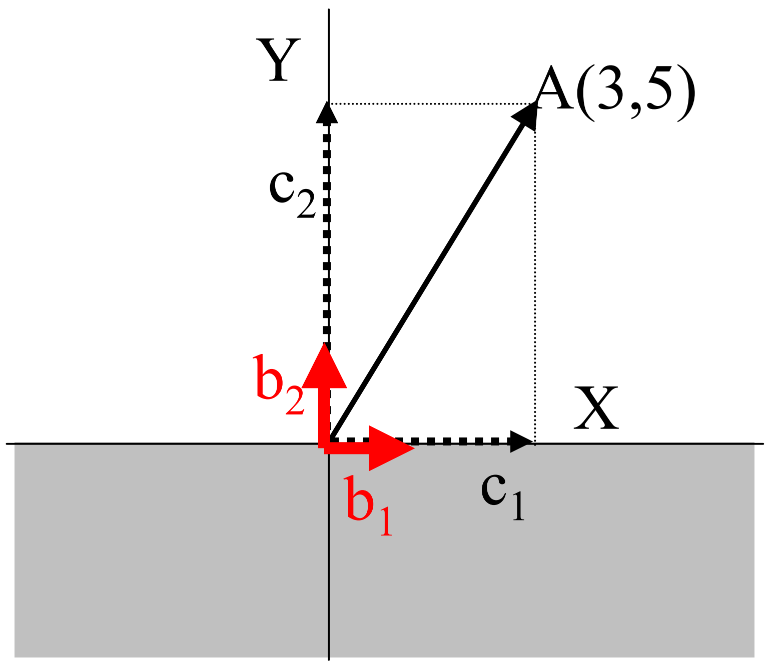
Vector decomposition



$$A = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

Let's Decompose into C_1 and C_2 !

Vector decomposition



Unit vectors

$$X = b_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad Y = b_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Calculating inner product !

$$c_1 = b_1^T A = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \end{pmatrix} = 3$$

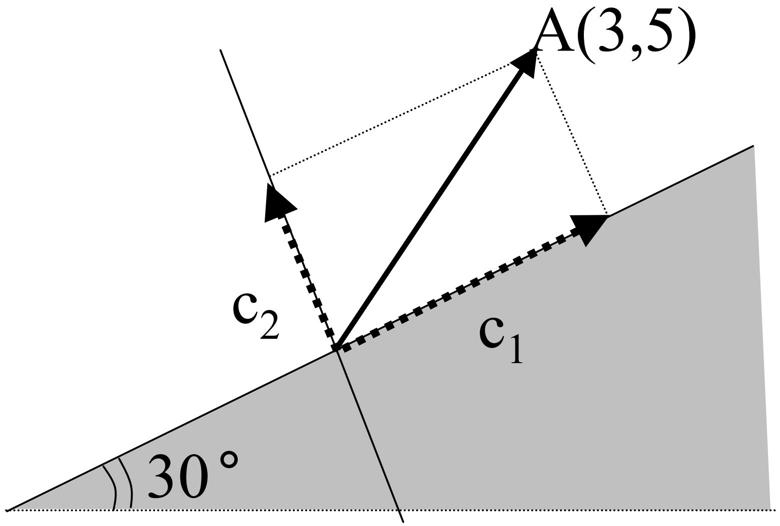
$$c_2 = b_2^T A = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \end{pmatrix} = 5$$

$$\Downarrow$$

$$C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = B^T A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

Vector decomposition

Vector decomposition with slope



Unit vectors

$$b_1 = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix} \qquad b_2 = \begin{pmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$$

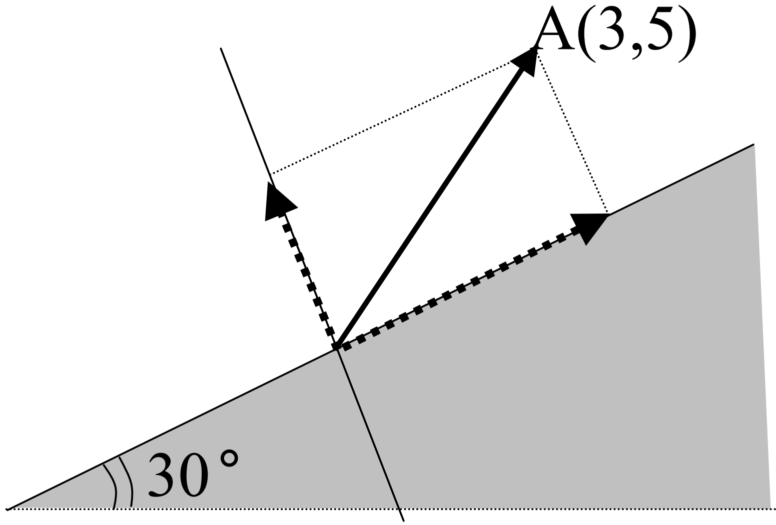
Remember! Inner product is 0

$$b_1 \cdot b_2 = \frac{\sqrt{3}}{2} \times \frac{-1}{2} + \frac{1}{2} \times \frac{\sqrt{3}}{2} = 0$$

$$C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = B^T A = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} \frac{3\sqrt{3}+5}{2} \\ \frac{5\sqrt{3}-3}{2} \end{pmatrix} = \begin{pmatrix} 5.098 \\ 2.830 \end{pmatrix}$$

Vector decomposition

Vector decomposition with slope



Inverse transform

B and C ⟶ A

$$A = \left(B^T\right)^{-1} C = BC = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} \frac{3\sqrt{3}+5}{2} \\ \frac{5\sqrt{3}-3}{2} \end{pmatrix} = \begin{pmatrix} \frac{9+5\sqrt{3}-5\sqrt{3}+3}{4} \\ \frac{3\sqrt{3}+5+15-3\sqrt{3}}{4} \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

Vector decomposition and orthogonal transform

Orthogonal transform

$$C = B^T A$$

$$C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = B^T A = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} \frac{3\sqrt{3}+5}{2} \\ \frac{5\sqrt{3}-3}{2} \end{pmatrix} = \begin{pmatrix} 5.098 \\ 2.830 \end{pmatrix}$$

Inverse transform

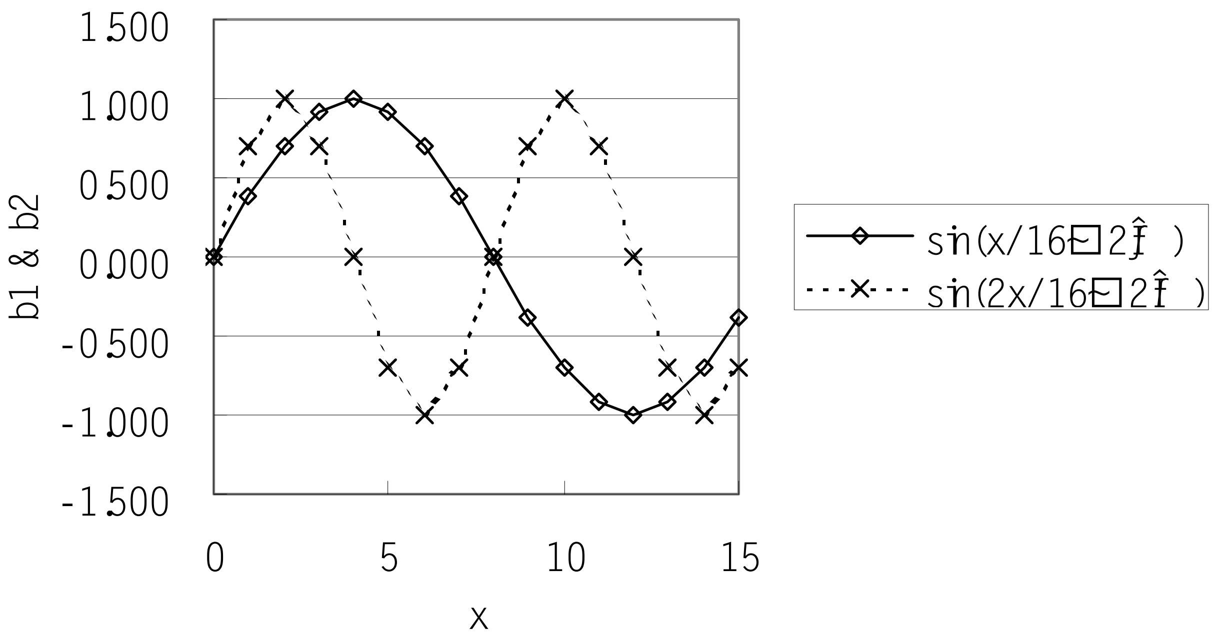
$$A = BC$$

$$A = \left(B^T\right)^{-1} C = BC = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} \frac{3\sqrt{3}+5}{2} \\ \frac{5\sqrt{3}-3}{2} \end{pmatrix} = \begin{pmatrix} \frac{9+5\sqrt{3}-5\sqrt{3}+3}{4} \\ \frac{3\sqrt{3}+5+15-3\sqrt{3}}{4} \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

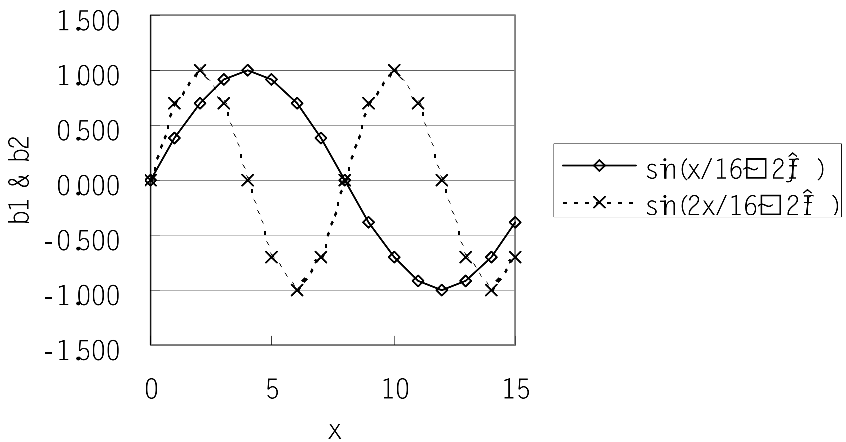
Trigonometric functions are orthogonal

Two trigonometric functions (Two vectors)

$$b_1 = \sin\left(1 \times \frac{x}{16} \times 2\pi\right) \qquad b_2 = \sin\left(2 \times \frac{x}{16} \times 2\pi\right)$$



Trigonometric functions are orthogonal



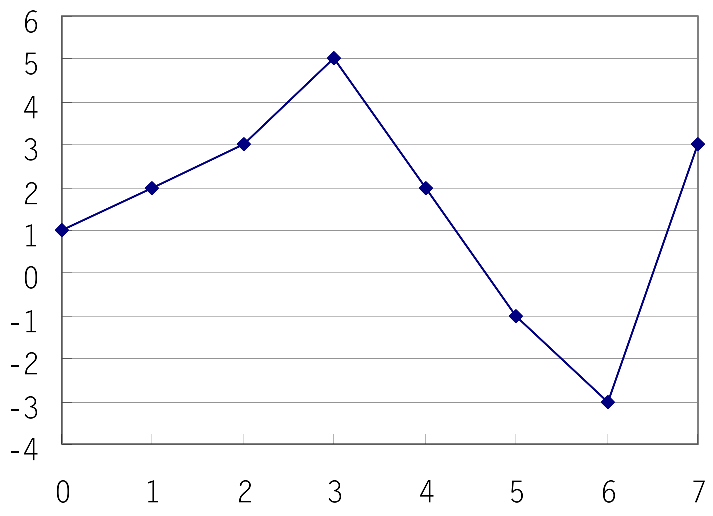
Inner product

x	$b_1=\sin(1\times x/16\times 2\pi)$	$b_2=\sin(2\times x/16\times 2\pi)$	$b_1\times b_2$
0	0.000	0.000	0.000
1	0.383	0.707	0.271
2	0.707	1.000	0.707
3	0.924	0.707	0.653
4	1.000	0.000	0.000
5	0.924	-0.707	-0.653
6	0.707	-1.000	-0.707
7	0.383	-0.707	-0.271
8	0.000	0.000	0.000
9	-0.383	0.707	-0.271
10	-0.707	1.000	-0.707
11	-0.924	0.707	-0.653
12	-1.000	0.000	0.000
13	-0.924	-0.707	0.653
14	-0.707	-1.000	0.707
15	-0.383	-0.707	0.271
		Total	0.000

Discrete Fourier transform

Let's transform 8 samples discrete waveform data!

Waveform



Vector notation

$$A = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 5 \\ 2 \\ -1 \\ -3 \\ 3 \end{pmatrix}$$

Discrete Fourier transform

Matrix consists of 8 trigonometric functions

$$B_{ij} = \begin{pmatrix} \sin\left(0\times\frac{0}{8}\times 2\pi\right) & \sin\left(1\times\frac{0}{8}\times 2\pi\right) & \cdot & \cdot & \cdot & \cdot & \cdot & \sin\left(7\times\frac{0}{8}\times 2\pi\right) \\ \sin\left(0\times\frac{1}{8}\times 2\pi\right) & \cdot & & & & & & \cdot \\ \sin\left(0\times\frac{2}{8}\times 2\pi\right) & & \cdot & & & & & \cdot \\ \cdot & & & \cdot & & & & \cdot \\ \cdot & & & & \cdot & & & \cdot \\ \cdot & & & & & \cdot & & \cdot \\ \cdot & & & & & & \cdot & \cdot \\ \sin\left(0\times\frac{7}{8}\times 2\pi\right) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sin\left(7\times\frac{7}{8}\times 2\pi\right) \end{pmatrix} = \left(\sin\left(n\times\frac{x}{8}\times 2\pi\right)\right)$$

Discrete Fourier transform

$$B_{ij} = \begin{pmatrix} \sin\left(0\times\frac{0}{8}\times 2\pi\right) & \sin\left(1\times\frac{0}{8}\times 2\pi\right) & \cdot & \cdot & \cdot & \cdot & \cdot & \sin\left(7\times\frac{0}{8}\times 2\pi\right) \\ \sin\left(0\times\frac{1}{8}\times 2\pi\right) & \cdot & & & & & & \cdot \\ \sin\left(0\times\frac{2}{8}\times 2\pi\right) & & \cdot & & & & & \cdot \\ \cdot & & & \cdot & & & & \cdot \\ \cdot & & & & \cdot & & & \cdot \\ \cdot & & & & & \cdot & & \cdot \\ \cdot & & & & & & \cdot & \cdot \\ \sin\left(0\times\frac{7}{8}\times 2\pi\right) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sin\left(7\times\frac{7}{8}\times 2\pi\right) \end{pmatrix} = \left(\sin\left(n\times\frac{x}{8}\times 2\pi\right)\right)$$

Extract n×x

$$W_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 \\ 0 & 3 & 6 & 9 & 12 & 15 & 18 & 21 \\ 0 & 4 & 8 & 12 & 16 & 20 & 24 & 28 \\ 0 & 5 & 10 & 15 & 20 & 25 & 30 & 35 \\ 0 & 6 & 12 & 18 & 24 & 30 & 36 & 42 \\ 0 & 7 & 14 & 21 & 28 & 35 & 42 & 49 \end{pmatrix} = (n\times x)$$

Discrete Fourier transform

Real part (cos)

$$B_{\mathrm{Re}} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0.71 & 0 & -0.71 & -1 & -0.71 & 0 & 0.71 \\ 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \\ 1 & -0.71 & 0 & 0.71 & -1 & 0.71 & 0 & -0.71 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -0.71 & 0 & 0.71 & -1 & 0.71 & 0 & -0.71 \\ 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \\ 1 & 0.71 & 0 & -0.71 & -1 & -0.71 & 0 & 0.71 \end{pmatrix} = \cos\left(\frac{1}{8} \times 2\pi \times W_{ij}\right)$$

Imaginary part (cos)

$$B_{\mathrm{Im}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.71 & -1 & -0.71 & 0 & 0.71 & 1 & 0.71 \\ 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & -0.71 & 1 & -0.71 & 0 & 0.71 & -1 & 0.71 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.71 & -1 & 0.71 & 0 & -0.71 & 1 & -0.71 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 \\ 0 & 0.71 & 1 & 0.71 & 0 & -0.71 & -1 & -0.71 \end{pmatrix} = -\sin\left(\frac{1}{8} \times 2\pi \times W_{ij}\right)$$

Discrete Fourier transform

Fourier transform is

$$A = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 5 \\ 2 \\ -1 \\ -3 \\ 3 \end{pmatrix}$$

$$C_{\mathrm{Re}} = B_{\mathrm{Re}}^{T} A$$

$$C_{\mathrm{Im}} = B_{\mathrm{Im}}^{T} A$$

$$C_{\mathrm{Re}} = \begin{pmatrix} 12.00 \\ -0.29 \\ 3.00 \\ -1.71 \\ -6.00 \\ -1.71 \\ 3.00 \\ -0.29 \end{pmatrix}$$

$$C_{\mathrm{Im}} = \begin{pmatrix} 0.00 \\ -9.54 \\ 7.00 \\ 2.46 \\ 0.00 \\ -2.46 \\ -7.00 \\ 9.54 \end{pmatrix}$$

Discrete Fourier transform

Inverse Fourier transform is

$$C_{\mathrm{Re}} = \begin{pmatrix} 12.00 \\ -0.29 \\ 3.00 \\ -1.71 \\ -6.00 \\ -1.71 \\ 3.00 \\ -0.29 \end{pmatrix}$$

$$A_{\mathrm{Re}} = \frac{1}{8}\left(B_{\mathrm{Re}} C_{\mathrm{Re}} - \left(-B_{\mathrm{Im}}\right) C_{\mathrm{Im}}\right)$$

$$A_{\mathrm{Re}} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 5 \\ 2 \\ -1 \\ -3 \\ 3 \end{pmatrix}$$

$$C_{\mathrm{Im}} = \begin{pmatrix} 0.00 \\ -9.54 \\ 7.00 \\ 2.46 \\ 0.00 \\ -2.46 \\ -7.00 \\ 9.54 \end{pmatrix}$$

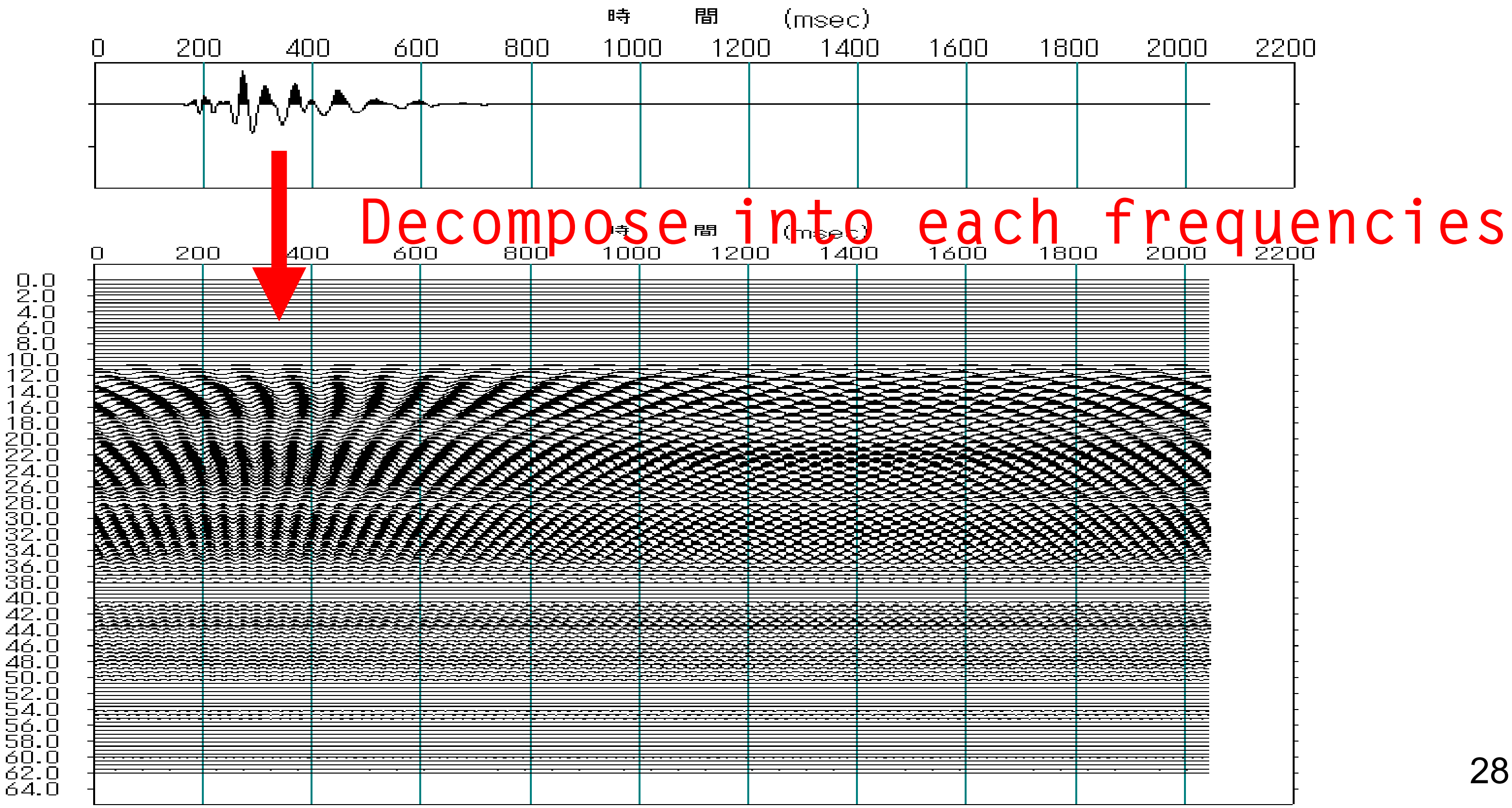
$$A_{\mathrm{Im}} = \frac{1}{8}\left(B_{\mathrm{Re}} C_{\mathrm{Im}} + \left(-B_{\mathrm{Im}}\right) C_{\mathrm{Re}}\right)$$

$$A_{\mathrm{Im}} = 0$$

Fourier series

Any waveform data can be decomposed into trigonometric functions!

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2\pi n t}{T} + b_n \sin \frac{2\pi n t}{T} t \right)$$



Fourier transform

Calculating Fourier coffients□a_0, a_1 to a_k, $b_1$to b_k□

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \tfrac{2\pi nt}{T} + b_n \sin \tfrac{2\pi nt}{T} t \right)$$

Fourier transform

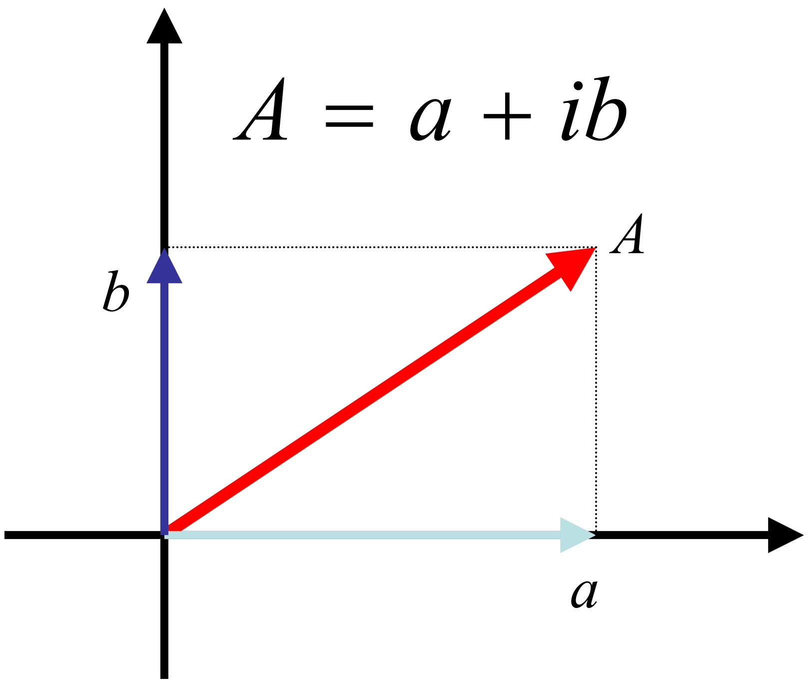
$$F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(t) \cdot \exp^{-i\omega t} dt$$

Inverse Fourier transform

$$f(t) = \int_{-\infty}^{+\infty} F(\omega) \cdot \exp^{-i\omega t} d\omega$$

Complex numbers

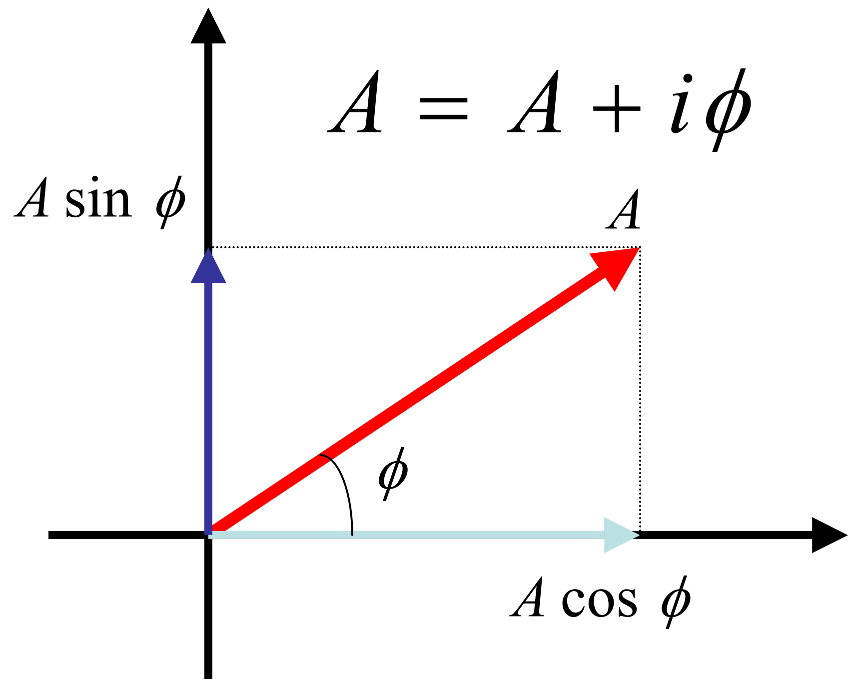
Complex plane with cos and sin



$$A = a\cos\frac{2\pi t}{T} + b\sin\frac{2\pi t}{T}$$

$$A = \sqrt{a^2 + b^2}$$

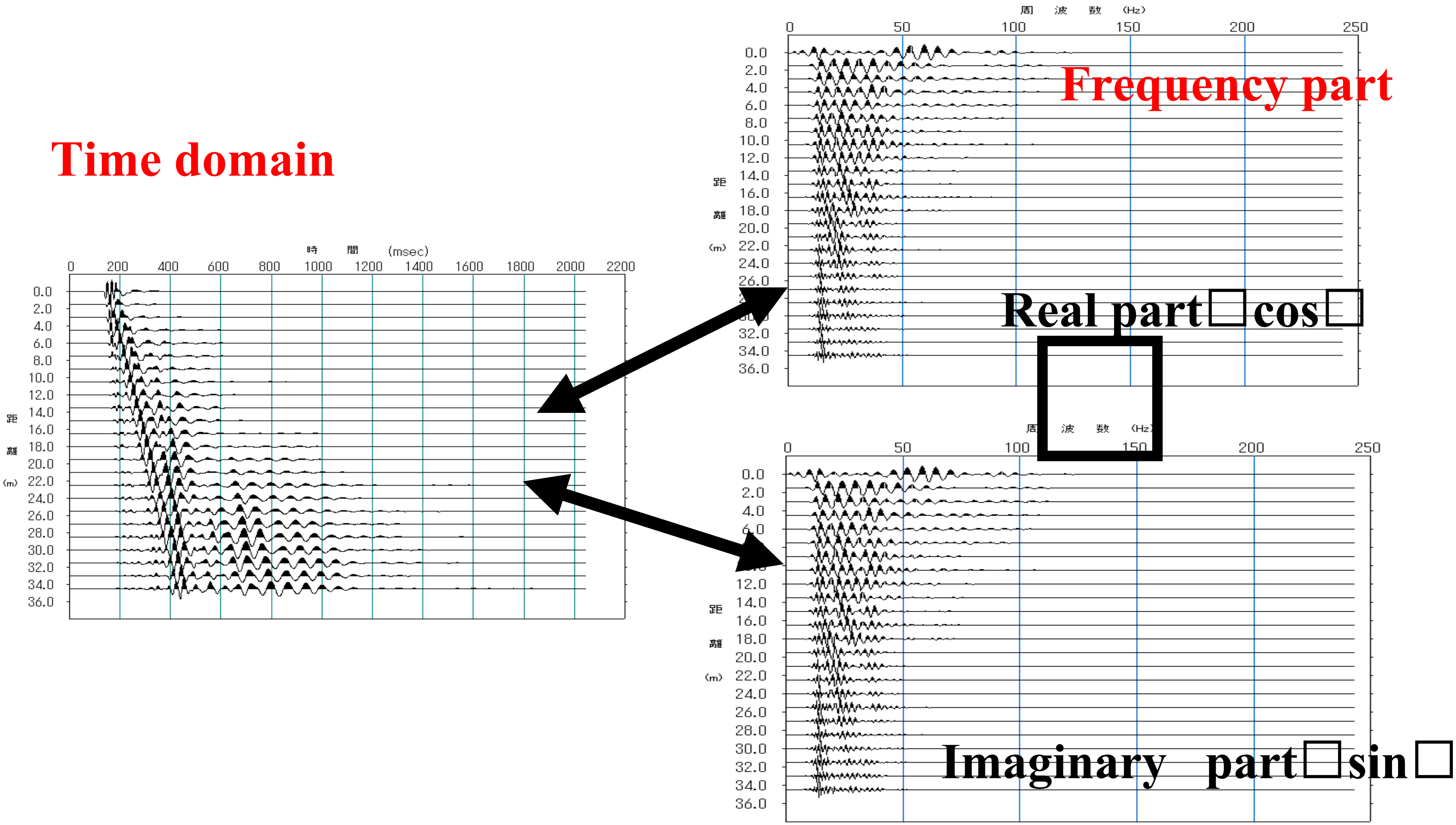
Complex plane with amplitude (A) and phase (Φ)



$$A = A\cos\phi + iA\sin\phi$$

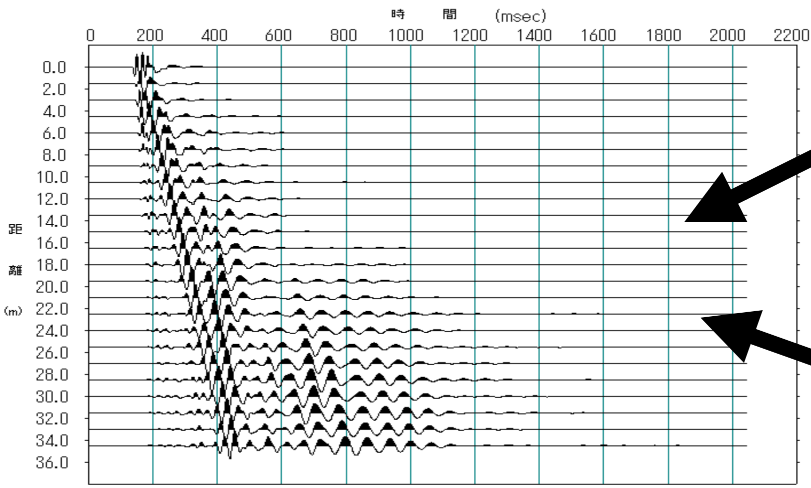
$$\phi = \tan^{-1}\frac{b}{a}$$

Complex numbers

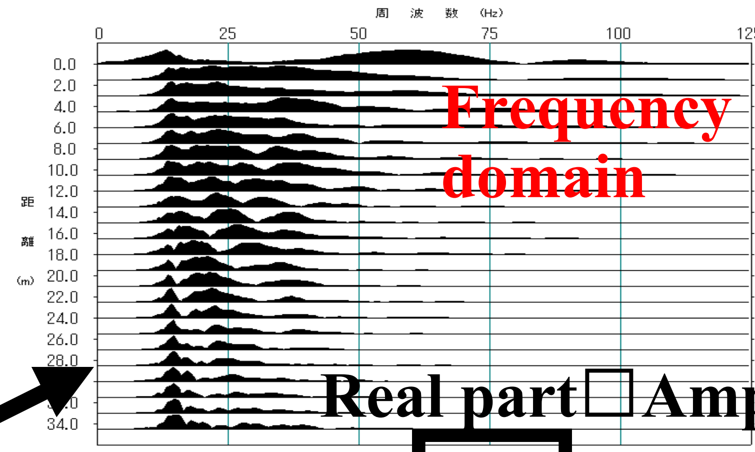


Amplitude and phase

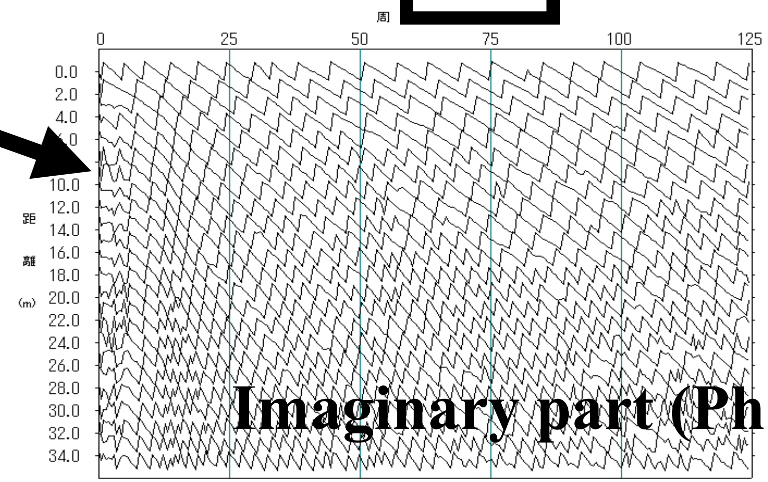
Time domain



Frequency domain



Real part□Amplitude□



Imaginary part (Phase)

Phase Velocity and Dispersion Curves

- Introduction to Surface Waves
- Fourier Transform
- Phase Velocity and Dispersion Curves
- Inversion
- Active Method
- Passive Method
- Application to Engineering Problems

Phase-velocity calculation

- Phase-difference
- Cross correlation
- τ-p transform in time domain
- τ-p transform in Frequency domain (MASW)
- Spatial auto correlation (SPAC)

Phase velocity calculation
Phase difference

Calculate the phase difference between two waves; *f(t)* and *g(t)*.

Fourier transform

$$F(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} f(t)\cdot \exp^{-i\omega t} dt \qquad G(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} g(t)\cdot \exp^{-i\omega t} dt$$

Amplitude and phase

$$F(\omega)= A_f(\omega)\cdot \exp^{-i\phi_f(\omega)} \qquad G(\omega)= A_g(\omega)\cdot \exp^{-i\phi_g(\omega)}$$

Phase difference

$$\Delta\phi(\omega)=\phi_f(\omega)-\phi_g(\omega)$$

Phase velocity□*c(ω)*□

$$c(\omega)=\frac{\omega\cdot\Delta x}{\Delta\phi_f(\omega)}$$

$$\boxed{c(\omega)=\frac{\omega\cdot\Delta x}{\Delta\phi_f(\omega)+2n\pi}}$$

Phase velocity calculation
Cross-correlation

Fourier transform

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(t)\cdot \exp^{-i\omega t} dt \qquad G(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} g(t)\cdot \exp^{-i\omega t} dt$$

Cross-correlation

$$CC_{fg}(\omega) = F(\omega)\cdot \overline{G(\omega)} = A_f(\omega)A_g(\omega)\cdot \exp^{i\Delta\phi(\omega)}$$

Phase velocity calculation

τ-p transform

τ-p transform

$$F(\tau, p) = \int_{-\infty}^{+\infty} f(x, t + xp)\,dx$$

$$p = \frac{1}{c} \qquad \tau = t$$

Fourier transform

$$F\left(\omega, \frac{1}{c}\right) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F\left(\tau, \frac{1}{c}\right) \cdot e^{-i\omega\tau}\, d\tau$$

Phase velocity calculation

τ-p transform in Frequency domain (MASW)

Observed waveform

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(t)\cdot e^{-i\omega t}\,dt$$

$$f(x,t) \xrightarrow{\text{Fourier transform}} F(x,\omega)$$

$$F(\tau,p) = \int_{-\infty}^{+\infty} f(x, t + xp)\,dx$$

τ-p transform
(Slant stack)

$$F(c,\omega) = \int_{-\infty}^{+\infty} F(x,\omega)\cdot e^{i\omega\frac{x}{c}}\,dx$$

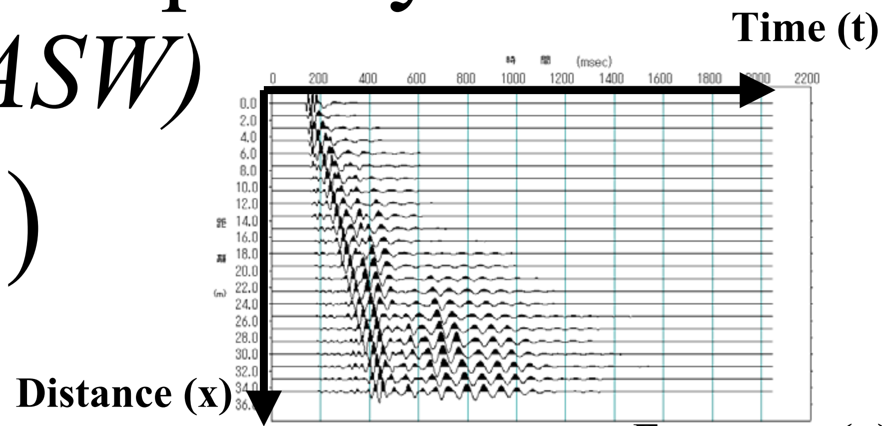
Phase shift and stack

$$F\left(\omega, \frac{1}{c}\right) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\tau,p)\cdot e^{-i\omega\tau}\,d\tau$$

$$F(\tau,p) \xrightarrow{\text{Fourier transform}} F(c,\omega)$$

Phase-velocity

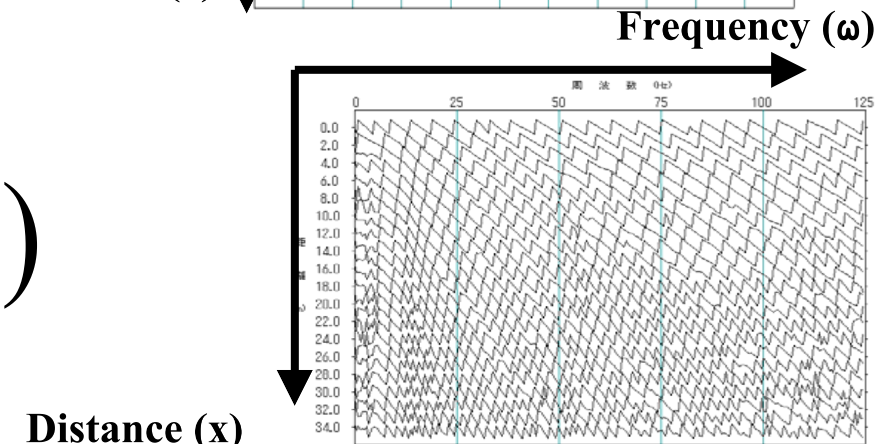
τ-p transform in Frequency domain *(MASW)*



$$f(x,t)$$

Fourier transform

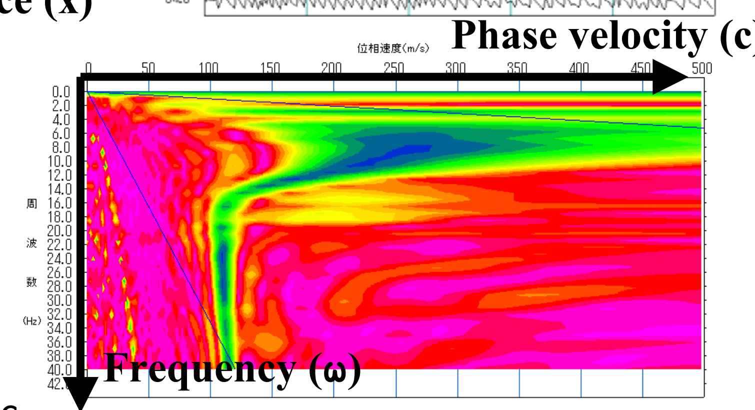
$$F(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} f(t)\cdot e^{-i\omega t}\,dt$$



$$F(x,\omega)$$

Phase shift□Stack

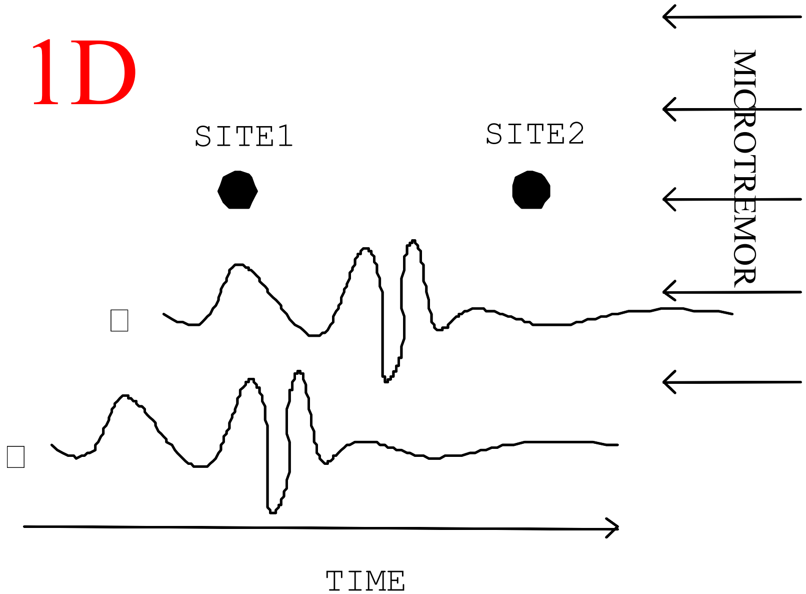
$$F(c,\omega)=\int_{-\infty}^{+\infty} F(x,\omega)\cdot e^{i\omega\frac{x}{c}}\,dx$$



$$F(c,\omega)$$

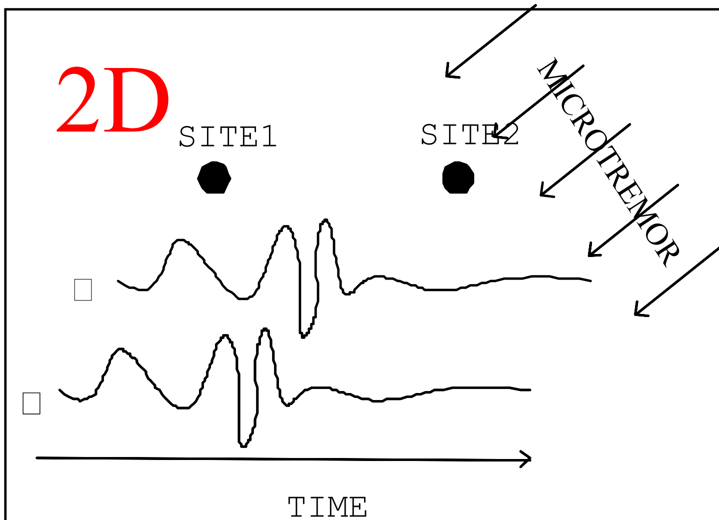
Spatial auto correlation (SPAC)

Calculating phase velocity from micro-tremors (passive method)



Microtremor propagate parallel to survey line

Phase velocity can be calculated directly



Microtremor propagate obliquely to survey line.

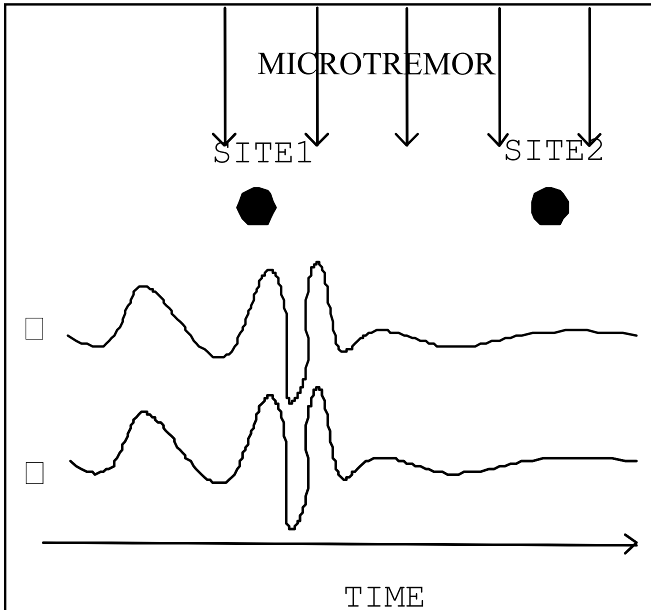
The input angle is known

-> phase velocity can be calculated.

The input angle isn't known

-> apparent velocity is higher than true phase velocity

Spatial auto correlation (2D)

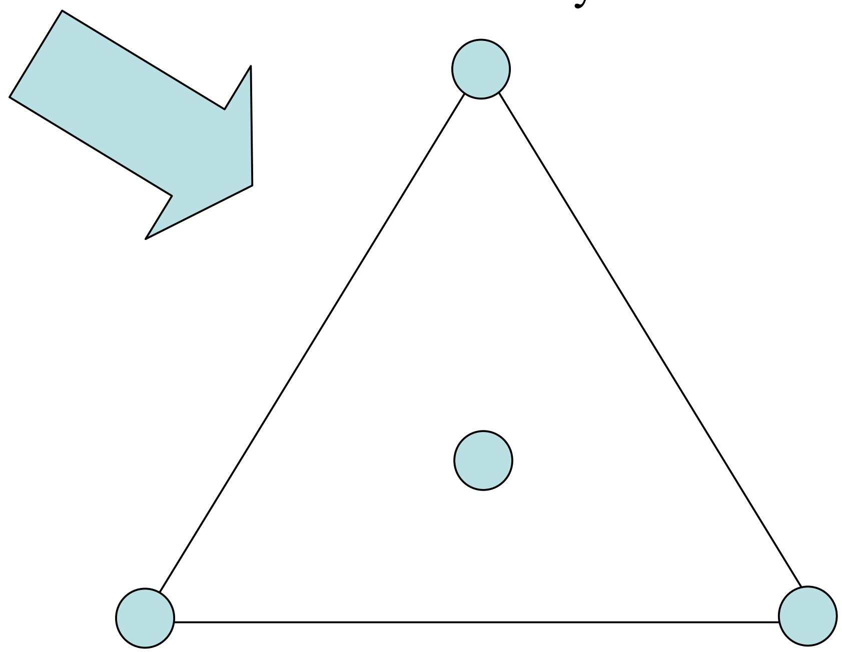


Microtremor propagate vertically to survey line
Phase velocity can’t be calculated.

We can’t know the direction of microtremors propagation before measuring.

And the sources of microtremors is studded.

So microtremors stationary stochastic wave.



Isotropic sensor arrangement is need.
Like triangle array

Spatial auto correlation (SPAC)

Cross-correlation

$$CC_{fg}(\omega) = F(\omega)\cdot\overline{G(\omega)} = A_f(\omega)A_g(\omega)\cdot\exp^{i\Delta\phi(\omega)}$$

Phase velocity□$c(\omega)$□

Substitute

$$c(\omega) = \frac{\omega\cdot\Delta x}{\Delta\phi(\omega)} \longrightarrow \Delta\phi(\omega) = \frac{\omega\cdot\Delta x}{c(\omega)}$$

$$CC_{fg}(\omega) = F(\omega)\cdot\overline{G(\omega)} = A_f(\omega)A_g(\omega)\cdot\exp^{i\frac{\omega\cdot\Delta x}{c(\omega)}}$$

$$CC_{fg}(\omega) = A_f(\omega)A_g(\omega)\cos\left(\frac{\omega\cdot\Delta x}{c(\omega)}\right)$$

Coherence

$$COH_{fg}(\omega) = \frac{CC_{fg}(\omega)}{A_f(\omega)A_g(\omega)} = \cos\left(\frac{\omega\cdot\Delta x}{c(\omega)}\right)$$

Spatial auto correlation (1D)

Spatial auto correlation

Time domain

$$cc(\Delta x, t) = f(x,t) * \overline{f(x+\Delta x, t)}$$

Two traces with Δx separation

Frequency domain

$$CC(\Delta x, \omega) = F(x,\omega) \cdot \overline{G(x+\Delta x, \omega)}$$

Coherence

$$COH(\Delta x, \omega) = \frac{CC(\Delta x, \omega)}{AC(x,\omega) AC(x+\Delta x, \omega)}$$

$$COH(\Delta x, \omega) = \cos\left(\frac{\omega}{c(\omega)} \Delta x\right)$$

Spatial auto correlation (2D)

$$cc(\Delta x, \Delta y, t) = f(x, y, t) * \overline{f(x + \Delta x, y + \Delta y, t)}$$

$$CC(\Delta x, \Delta y, \omega) = F(x, y, \omega) \cdot \overline{G(x + \Delta x, y + \Delta y, \omega)}$$

$$\Delta x = r \cos \varphi$$
$$\Delta y = r \sin \varphi$$

$$CC(r, \omega) = F(x, y, \omega) \cdot \overline{G(x + \Delta x, y + \Delta y, \omega)}$$

$$COH(r, \omega) = \frac{CC(r, \omega)}{AC(x, y, \omega) AC(x + \Delta x, y + \Delta y, \omega)}$$

$$COH(r, \omega) = J_0\left(\frac{\omega}{c(\omega)} r\right)$$ → **Bessel function**

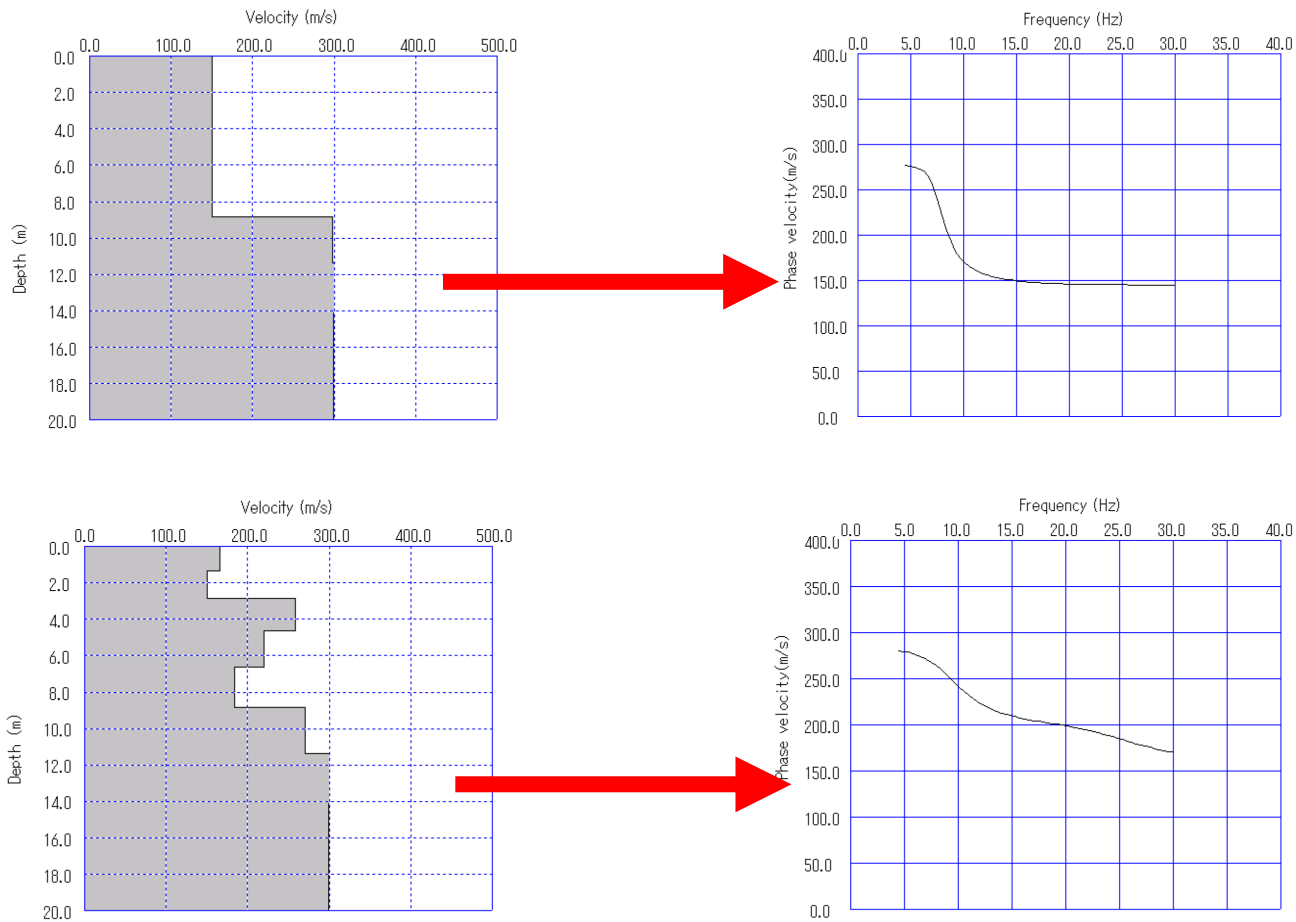
Characteristics of phase-velocity curves (dispersion curves)

- Phase-velocity curve is to be a smooth curved line, or a straight line.
- Phase-velocity curve reflects the averaged velocity model beneath receiver array.
- Higher modes exist.
- The frequency range is to be fixed by the minimum / maximum receiver spacing.

Characteristics of Phase-velocity curve

Phase velocity curve is to be a smooth curved line, or a straight line.

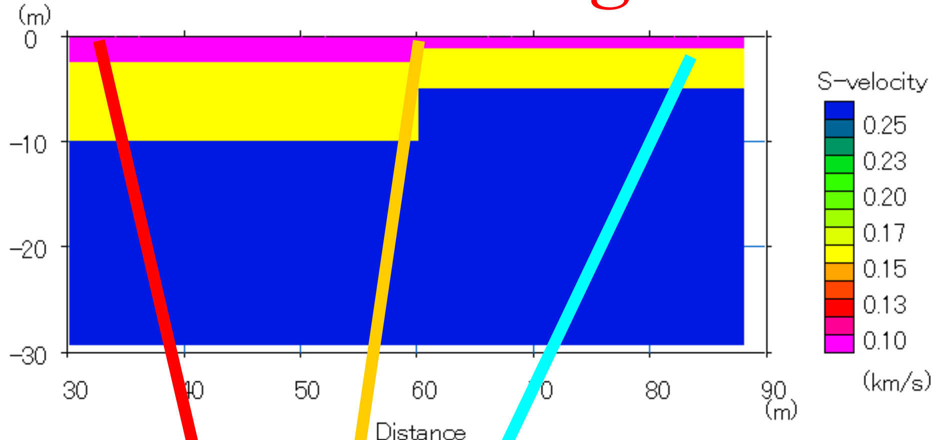
S-wave velocity structure **Phase velocity curve**

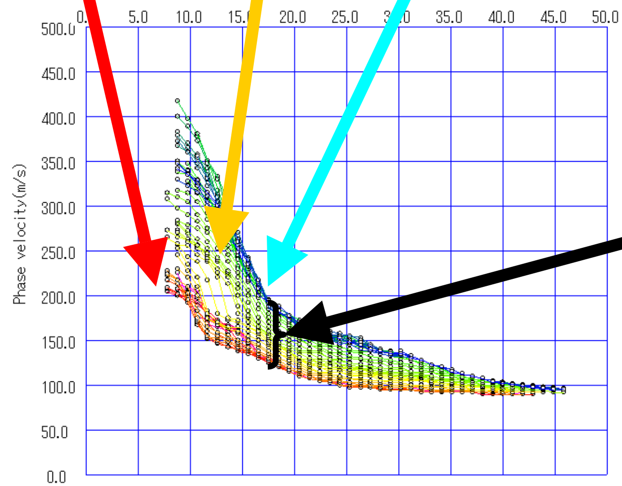


Characteristics of phase-velocity curve

Phase velocity curve reflects the averaged velocity model beneath receiver array.

Can not change drastically





Changes gradually

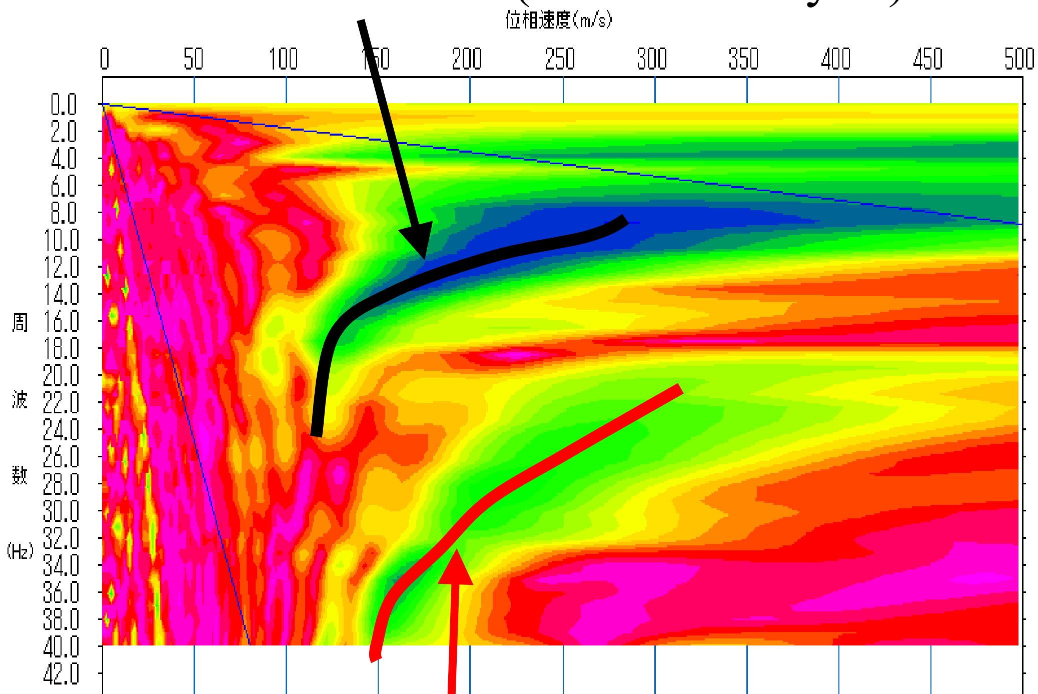
Characteristics of phase velocity curve

Higher mode

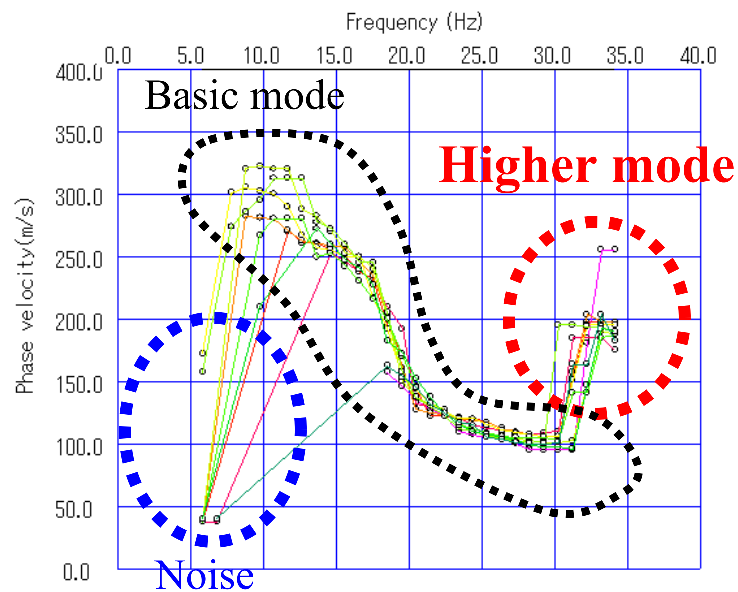
Wave form data including higher mode (Frequency domain)

Dispersion curve including noise and higher mode

Fundamental mode (used for analysis)



Higher mode (no need for analysis)

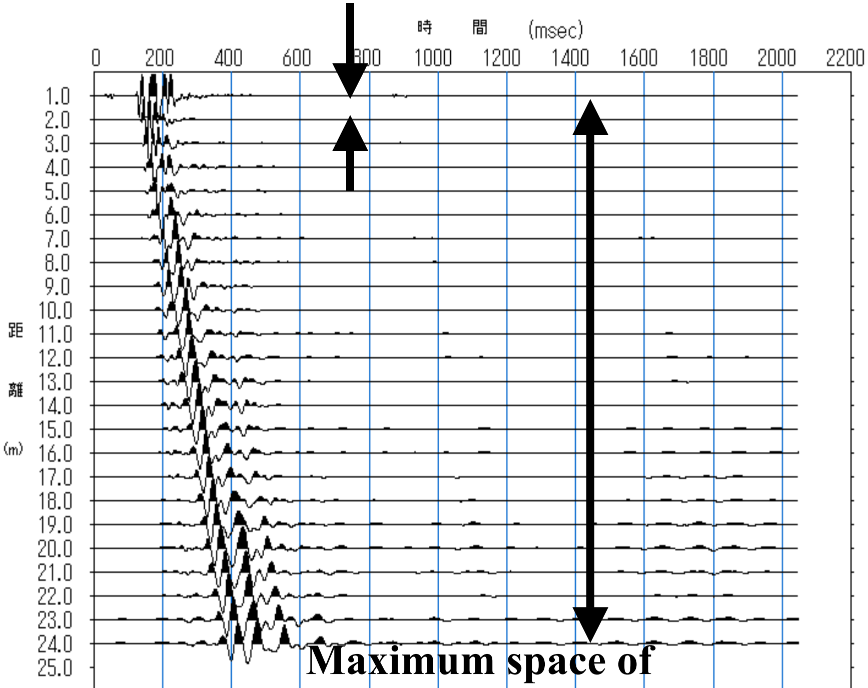


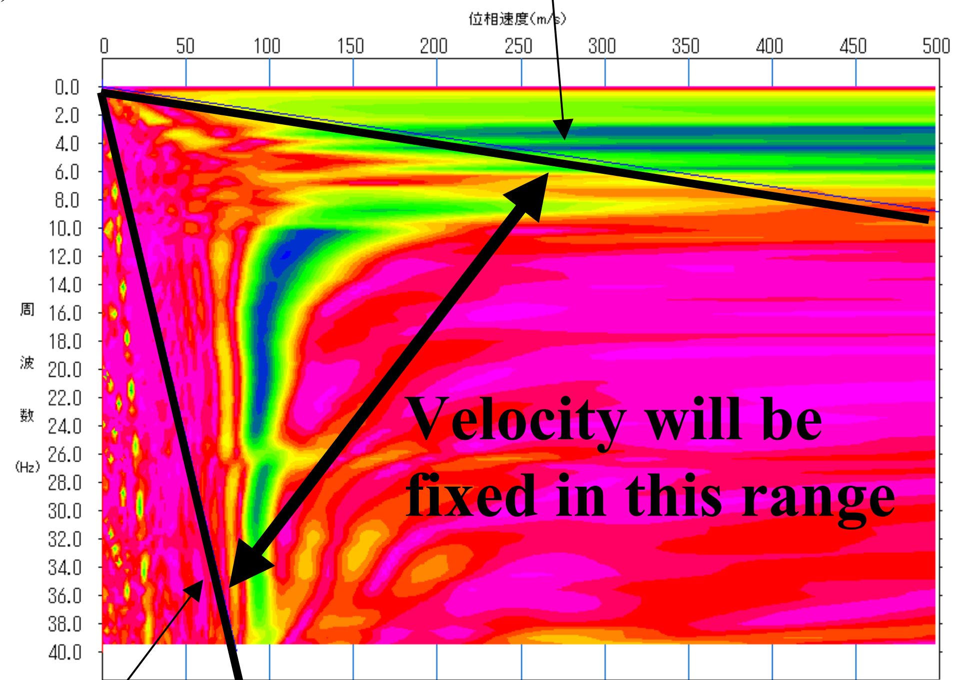
Characteristics of phase velocity curve

The frequency range is to be fixed by the minimum / maximum space of receivers.

Minimum space of receivers (1m)

The line that the wave length will be double of maximum space (46m) of receivers (It is not precise enough to determine the velocity on upper side from this line.)





Maximum space of receivers (23m)

The line that the wave length will be double of minimum space (2m) of receivers It is not enough to determine the velocity on left side from this line.)

Inversion

- Introduction to Surface Waves
- Fourier Transform
- Phase Velocity and Dispersion Curves
- Inversion
- Active Method
- Passive Method
- Application to Engineering Problems

Least Square Method

Simultaneous equations(Q1)

$$2x_1 + x_2 = 11$$
$$4x_1 + x_2 = 17$$
$$6x_1 + x_2 = 23$$

3 data for 2 unknown

Matrix notation

$$AX = \begin{pmatrix} 2 & 1 \\ 4 & 1 \\ 6 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 11 \\ 17 \\ 23 \end{pmatrix} = Y$$

Partial difference !

$$\boxed{\frac{\partial}{\partial x_1}(6x_1 + x_2 - 23) = 6}$$

$$AX = Y$$

Matrix A(Jacobian matrix)

$$f_1 = 2x_1 + x_2 - 11$$
$$f_2 = 4x_1 + x_2 - 17$$
$$f_3 = 6x_1 + x_2 - 23$$

$$\Downarrow$$

$$A = \begin{pmatrix} 2 & 1 \\ 4 & 1 \\ 6 & 1 \end{pmatrix} = \begin{pmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} \\ \dfrac{\partial f_3}{\partial x_1} & \dfrac{\partial f_3}{\partial x_2} \end{pmatrix}$$

Errors:e

$$e = AX - Y$$

Minimize the sum of squares of errors

$$E = (AX - Y)^T (AX - Y) = \|AX - Y\|^2 \Rightarrow Minimize$$

Set differential of E to zero

$$\frac{dE}{dX} = 2A^T (AX - Y) = 0$$

Solve for X

$$\boxed{(A^T A)X = A^T Y}$$

Normal equation

Example(Q1)

$$2x_1 + x_2 = 11$$
$$4x_1 + x_2 = 17$$
$$6x_1 + x_2 = 23$$

$$\Downarrow$$

$$\left(A^T A\right)X = \begin{pmatrix} 2 & 4 & 6 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 4 & 1 \\ 6 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 & 4 & 6 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 11 \\ 17 \\ 23 \end{pmatrix} = A^T Y$$

$$\Downarrow$$

$$\left(A^T A\right)X = \begin{pmatrix} 56 & 12 \\ 12 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 228 \\ 51 \end{pmatrix} = A^T Y$$ Normal equation

$$\Downarrow$$

$$X = \left(A^T A\right)^{-1} A^T Y = \begin{pmatrix} 0.125 & -0.5 \\ -0.5 & 2.3333 \end{pmatrix}\begin{pmatrix} 228 \\ 51 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

Non-linear least square method

If the Jacobian matrix is not a constant,

$$y(Z) = x_1 Z - x_2 e^{-Zx_3}$$

$$A = \begin{pmatrix} \dfrac{\partial y(z_1)}{\partial x_1} & \dfrac{\partial y(z_1)}{\partial x_2} & \dfrac{\partial y(z_1)}{\partial x_3} \\ \dfrac{\partial y(z_2)}{\partial x_1} & \dfrac{\partial y(z_2)}{\partial x_2} & \dfrac{\partial y(z_2)}{\partial x_3} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \dfrac{\partial y(z_m)}{\partial x_1} & \dfrac{\partial y(z_m)}{\partial x_2} & \dfrac{\partial y(z_m)}{\partial x_3} \end{pmatrix} = \begin{pmatrix} z_1 & -e^{-Z_1 x_3} & -x_2 Z_1 e^{-Z_1 x_3} \\ z_2 & -e^{-Z_2 x_3} & -x_2 Z_2 e^{-Z_2 x_3} \\ & & \\ & & \\ z_m & -e^{-Z_m x_3} & -x_2 Z_m e^{-Z_m x_3} \end{pmatrix}$$

parameter x is in the matrix A!

Iterative solution of non-linear least square method

1 : Calculate theoretical value Y_0 for initial value X_0.

$$Y_0(Z) = Y(Z, X_0)$$

2 : Calculate residuals (ΔY) between theoretical value Y_0 and observed value Y.

$$\Delta Y = Y - Y_0$$

3 : Calculate correction value for X (ΔY) by the least square method.

$$\left(A^T A\right)\Delta X = A^T \Delta Y$$

4 : Calculate new estimate X_1.

$$X_1 = X_0 + \Delta X$$

5 : Return to step 1.

6 : Stop if the residuals are enough small.

Example(Q3)

Model $$y(Z) = x_1 Z - x_2 e^{-Zx_3}$$

True solution(answer) $$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

Eleven observed data

z	y(z)
0	-2
1	0.264241
2	1.729329
3	2.900426
4	3.963369
5	4.986524
6	5.995042
7	6.998176
8	7.999329
9	8.999753
10	9.999909

Partial differential
$$\frac{\partial y}{\partial x_1} = Z \qquad \frac{\partial y}{\partial x_2} = -e^{-Zx_3} \qquad \frac{\partial y}{\partial x_3} = x_2 Z e^{-Zx_3}$$

Initial model
$$X_0 = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$$

Jacobian matrix A

$$A_0 = \begin{pmatrix} \frac{\partial y(z_1)}{\partial x_1} & \frac{\partial y(z_1)}{\partial x_2} & \frac{\partial y(z_1)}{\partial x_3} \\ \frac{\partial y(z_2)}{\partial x_1} & \frac{\partial y(z_2)}{\partial x_2} & \frac{\partial y(z_2)}{\partial x_3} \\ . & . & . \\ . & . & . \\ \frac{\partial y(z_{11})}{\partial x_1} & \frac{\partial y(z_{11})}{\partial x_2} & \frac{\partial y(z_{11})}{\partial x_3} \end{pmatrix} = \begin{pmatrix} z_1 & -e^{-Z_1 x_3} & -x_2 Z_1 e^{-Z_1 x_3} \\ z_2 & -e^{-Z_2 x_3} & -x_2 Z_2 e^{-Z_2 x_3} \\ & & \\ & & \\ z_{11} & -e^{-Z_{11} x_3} & -x_2 Z_{11} e^{-Z_{11} x_3} \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & -0.1353352832 & 0.4060058497 \\ 2 & -0.0183156389 & 0.1098938333 \\ 3 & -0.0024787522 & 0.0223087696 \\ 4 & -0.0003354626 & 0.0040255515 \\ 5 & -0.0000453999 & 0.0006809989 \\ 6 & -0.0000061442 & 0.0001105958 \\ 7 & -0.0000008315 & 0.0000174621 \\ 8 & -0.0000001125 & 0.0000027008 \\ 9 & -0.0000000152 & 0.0000004112 \\ 10 & -0.0000000021 & 0.0000000618 \end{pmatrix}$$

Observed data

$$Y^T = (-2.0000 \quad 0.264241 \quad 1.729329 \quad 2.900426 \quad 3.963369 \quad 4.986524 \quad 5.995042 \quad 6.998176 \quad 7.999329 \quad 8.999753 \quad 9.999909)$$

Theoretical data for initial the model

$$Y_0^T = (-3.0000 \quad 1.5940 \quad 3.9451 \quad 5.9926 \quad 7.9990 \quad 9.9999 \quad 12.0000 \quad 14.0000 \quad 16.0000 \quad 18.0000 \quad 20.0000)$$

Residual vector $\Delta Y = Y_0 - Y$

$$\Delta Y_0^T = (-1.0000 \quad 1.3298 \quad 2.2157 \quad 3.0921 \quad 4.0356 \quad 5.0133 \quad 6.0049 \quad 7.0018 \quad 8.0007 \quad 9.0002 \quad 10.0001)$$

RMSE(Root Mean Square Error)

$$RMSE_0 = \sqrt{\frac{\Delta Y_0^T \Delta Y_0}{11}} = 5.9449$$

$$A_0^T A_0 = \begin{pmatrix} 385 & -0.181 & 0.71304 \\ -0.181 & 1.0187 & -0.057 \\ 0.71304 & -0.057 & 0.17743 \end{pmatrix} \qquad A_0^T \Delta Y_0 = \begin{pmatrix} 386.3 \\ 0.7702 \\ 0.8728 \end{pmatrix}$$

Solve $\left(A_0^T A_0\right)\Delta X_0 = A_0^T \Delta Y_0$ get $\Delta X_0 = \begin{pmatrix} 1.0016 \\ 1.0021 \\ 1.2162 \end{pmatrix}$

New estimated value for X (X_1) $X_1 = X_0 - \Delta X$

$$X_1 = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1.0016 \\ 1.0021 \\ 1.2162 \end{pmatrix} = \begin{pmatrix} 0.9984 \\ 1.9979 \\ 0.7838 \end{pmatrix}$$

Calculate residuals (RMSE) from new estimation of X (X_1)

$$RMSE_1 = \sqrt{\frac{\Delta Y_1^T \Delta Y_1}{11}} = 0.0793$$

In the 2nd calculation

$$A_1^T A_1 = \begin{pmatrix} 385 & -1.543 & 8.19332 \\ -1.543 & 1.2635 & -0.6652 \\ 8.19332 & -0.6652 & 2.02955 \end{pmatrix} \quad A_1^T \Delta Y_1 = \begin{pmatrix} -1.854 \\ 0.123 \\ -0.372 \end{pmatrix}$$

Correction is

$$\Delta X_1 = \begin{pmatrix} -0.001 \\ 0.002 \\ -0.179 \end{pmatrix}$$

Corrected model is

$$X_2 = \begin{pmatrix} 0.9984 \\ 1.9979 \\ 0.7838 \end{pmatrix} - \begin{pmatrix} -0.001 \\ 0.002 \\ -0.179 \end{pmatrix} = \begin{pmatrix} 0.9994 \\ 1.9959 \\ 0.9625 \end{pmatrix} \cong \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

Residuals are

$$RMSE_2 = \sqrt{\frac{\Delta Y_2^T \Delta Y_2}{11}} = 0.0122 \cong 0$$

Inversion in surface wave methods

S-velocity model

$$x^T = \left(Vs_1, Vs_2, \cdots, Vs_M\right)$$

Objective function

$$\sum_i^N \left(f_i^{obs} - f_i^{cal}\left(Vs_1, Vs_2, \cdots Vs_N\right)\right)^2 = \sum_i^N \left(f_i^{obs} - f_i^{cal}(x)\right)^2 \rightarrow \text{Minimize}$$

Inversion in surface wave methods

Now, $f_i = f_i^{cal}(x)$ *(i□□□N□N is the number of observed data)*

Jacobian matrix (a) goes to

$$a = \begin{pmatrix} \frac{\partial f_1}{\partial Vs_1} & \frac{\partial f_1}{\partial Vs_2} & . & \frac{\partial f_1}{\partial Vs_N} \\ \frac{\partial f_2}{\partial Vs_1} & \frac{\partial f_2}{\partial Vs_2} & . & \frac{\partial f_2}{\partial Vs_N} \\ \frac{\partial f_3}{\partial Vs_1} & \frac{\partial f_3}{\partial Vs_2} & . & \frac{\partial f_3}{\partial Vs_N} \\ . & . & . & . \\ \frac{\partial f_n}{\partial Vs_1} & \frac{\partial f_n}{\partial Vs_2} & . & \frac{\partial f_n}{\partial Vs_N} \end{pmatrix}$$

Unknown x is in the partial differentials !

Non-linear problem

Iteration

Inversion in surface wave methods

Residual vector y is

$$y = \begin{pmatrix} f_1^{obs} - f_i^{cal}(x) \\ f_2^{obs} - f_2^{cal}(x) \\ f_3^{obs} - f_3^{cal}(x) \\ . \\ f_N^{obs} - f_N^{cal}(x) \end{pmatrix}$$

Observed Calculated

Correction vector Δx is,

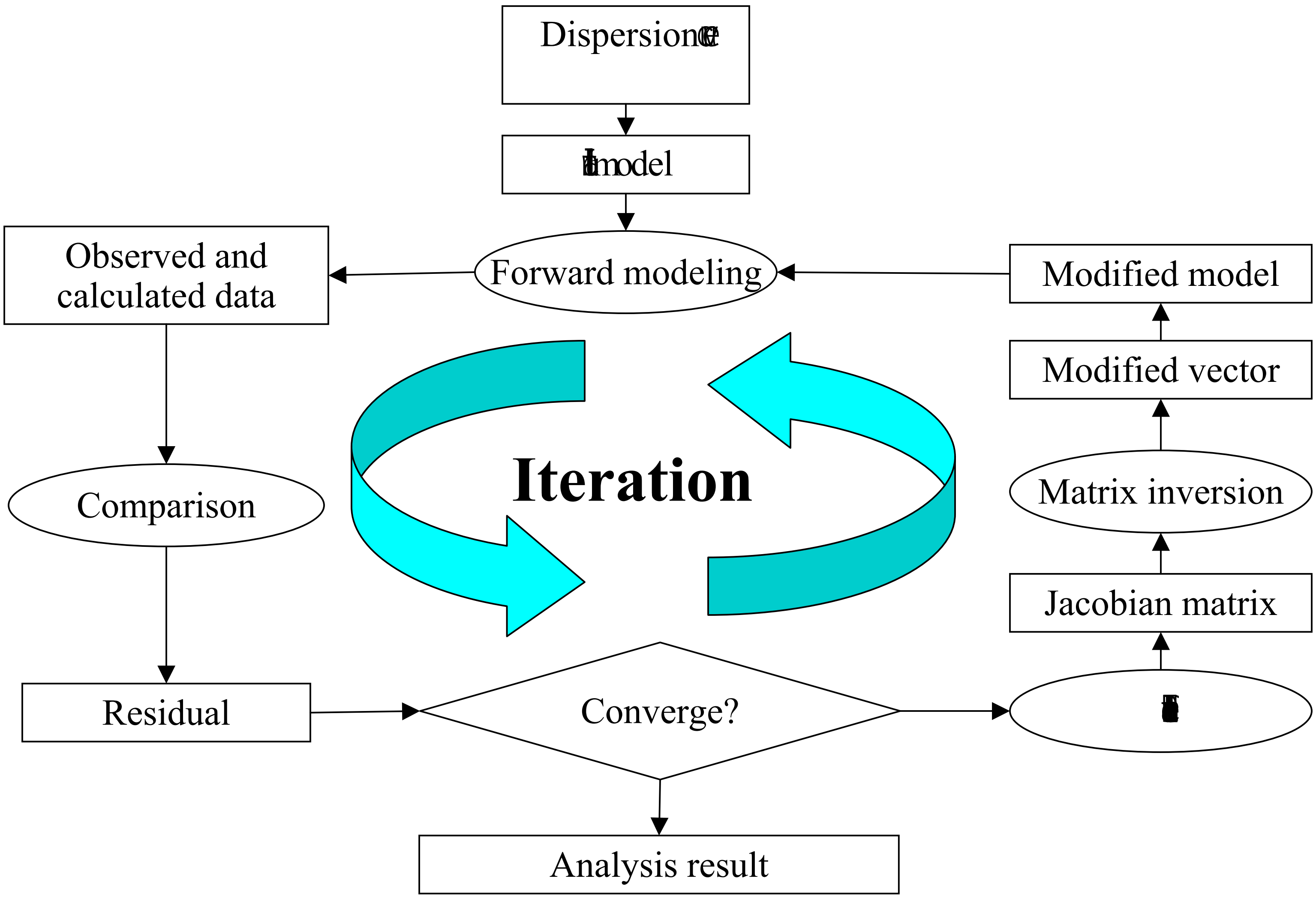
$$\left(a^T a + \varepsilon I\right)\Delta x = a^T y$$

Damping parameter

New model estimation x^{l+1} in l_{th} iteration is,

$$x^{l+1} = x^l + \gamma \Delta x$$

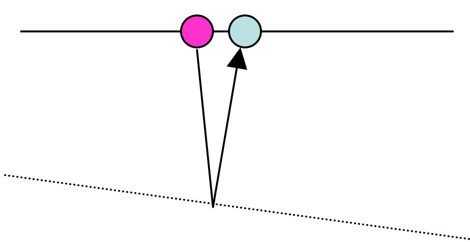
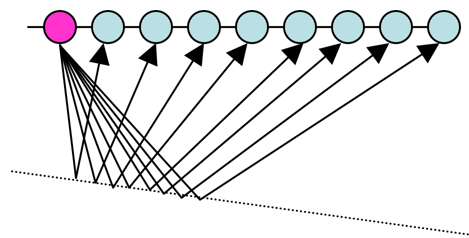
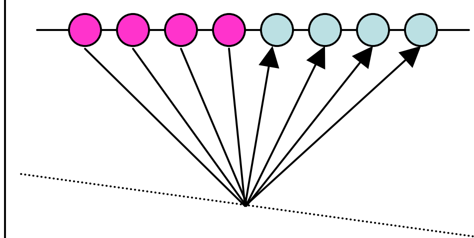
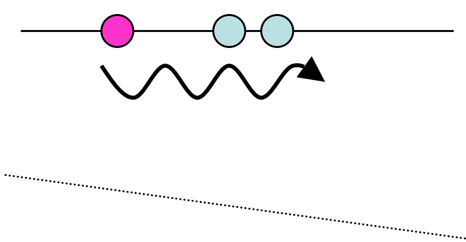
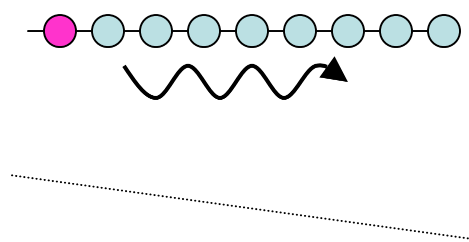
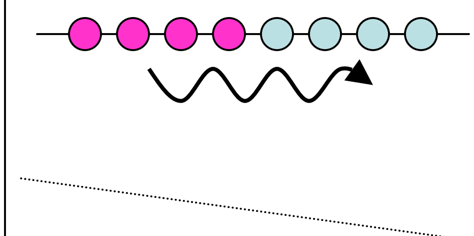
Inversion in surface wave methods



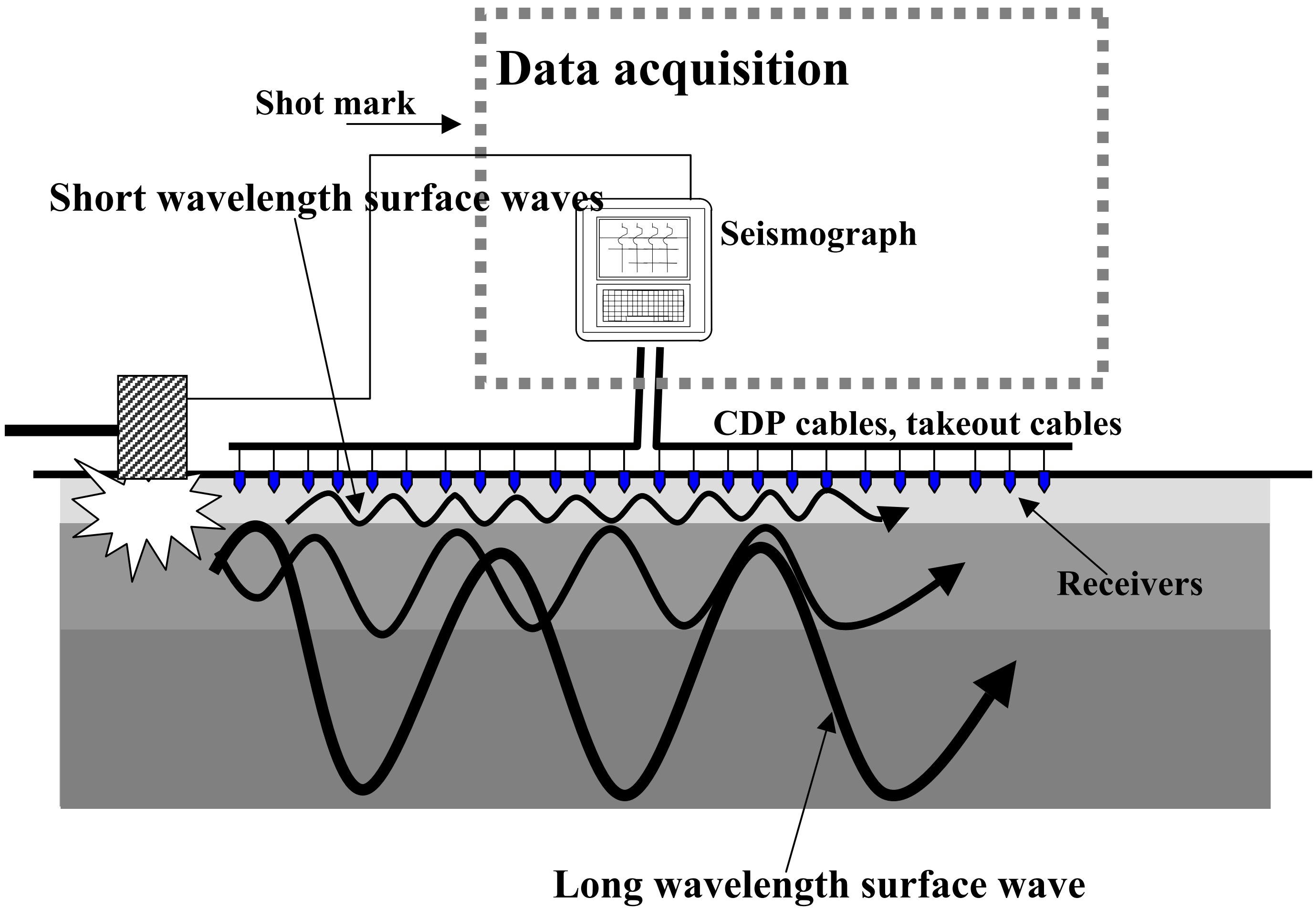
Active Method

- Introduction to Surface Waves
- Fourier Transform
- Phase Velocity and Dispersion Curve
- Inversion
- Active Method
- Passive Method
- Application to Engineering Problems

Seismic survey with reflection method and Surface wave

	Single channel	Shot gather	CDP gather
Reflection method			
	Old style for surfave wave analysis (SASW)	Multi-channel analysis (MASW)	CMP analysis
Surface wave			

Schematic diagram of data acquisition



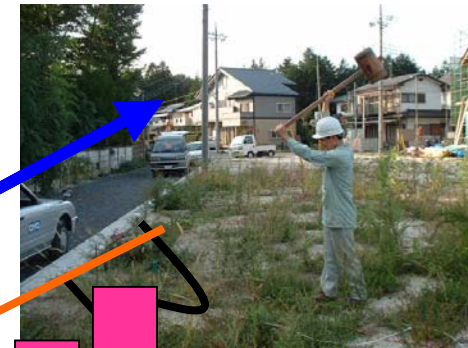
Data acquisition for the surface-wave method

Source : sledge hammer
Penetration depth□0 to 20m

Data acquisition□
20 minutes

Analysis:
15 minutes

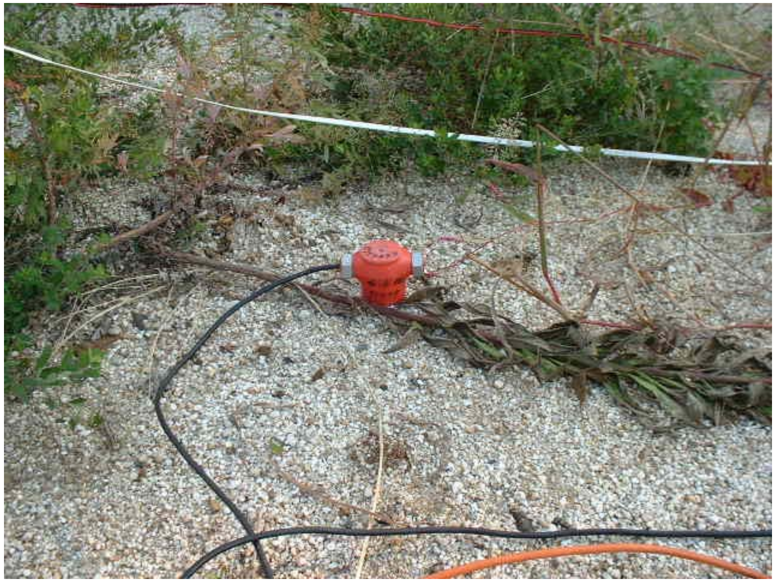
Survey line length :
20□30m



Receivers□1m spacing x 24

Sources (sledge hammer)□
1m spacing x 25 points

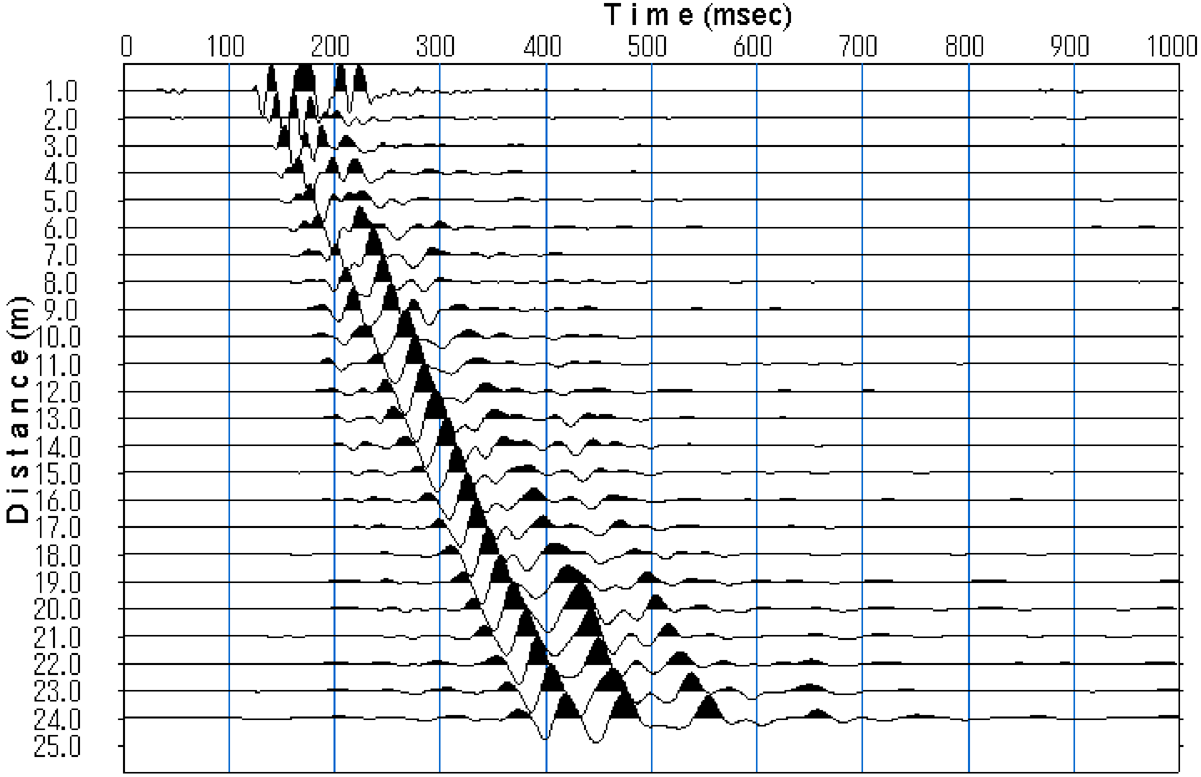
Data acquisition for the surface-wave method



Data example

Typical setting:
  Sampling rate : 1msec
  Data length : 1024

Do not saturate !
Keep all waveforms !



Source and receiver configuration for two-dimensional survey

In the two-dimensional surface-wave method, several source-receiver configurations can be used in data acquisition.

A)Fixed receivers

B)Continuous fixed receivers

C)End-on-spread

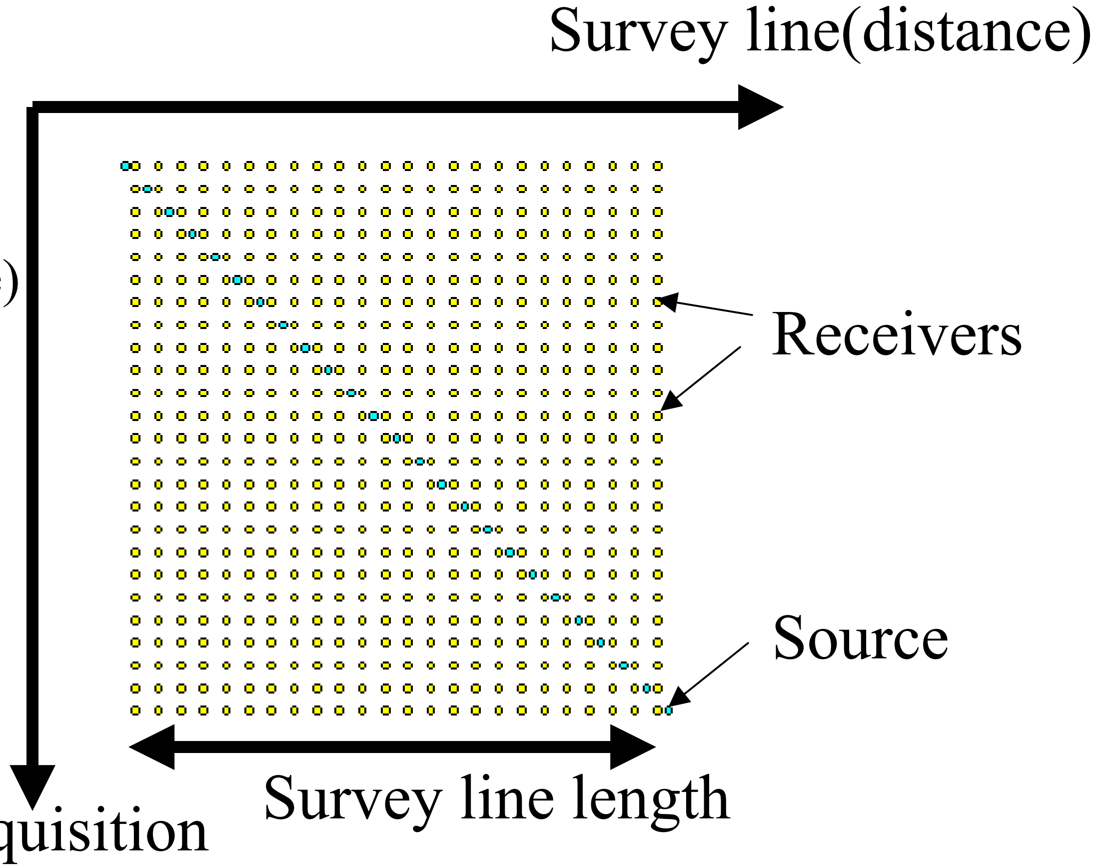
Source and receiver configuration for two-dimensional survey

A) Fixed receiver

Sources are placed in between receivers and both ends (outside) of survey line .

# of sources=# of receivers +1

At the both ends of survey line, accuracy of deeper part analysis result can be bad.

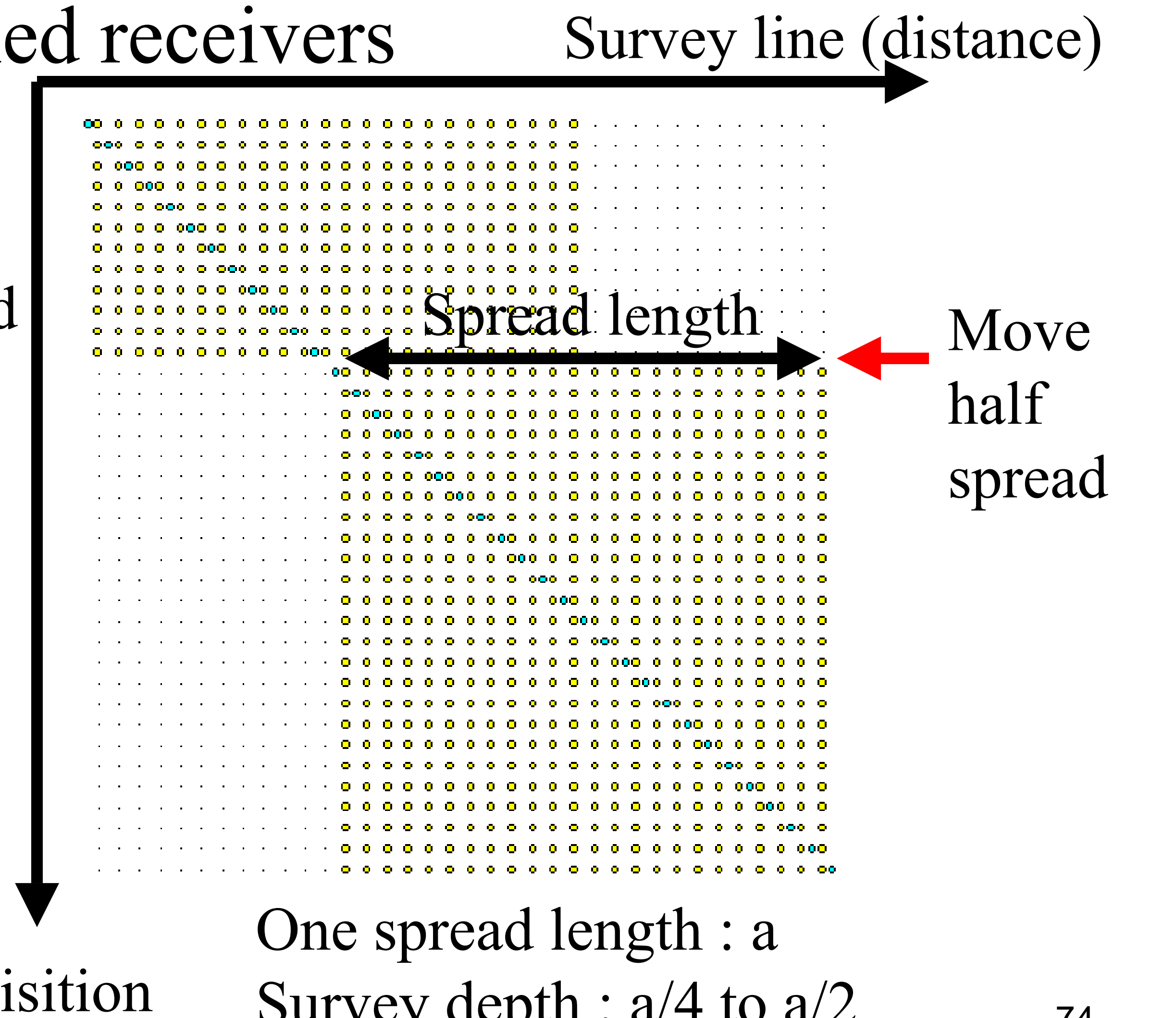


Survey line length : a
Survey depth : a/2

Source and receiver configuration for two-dimensional survey

B) Continuous fixed receivers

If source arrived at the middle of spread, first half of spread is moved to a new spread. This spread moving is performed continuously.

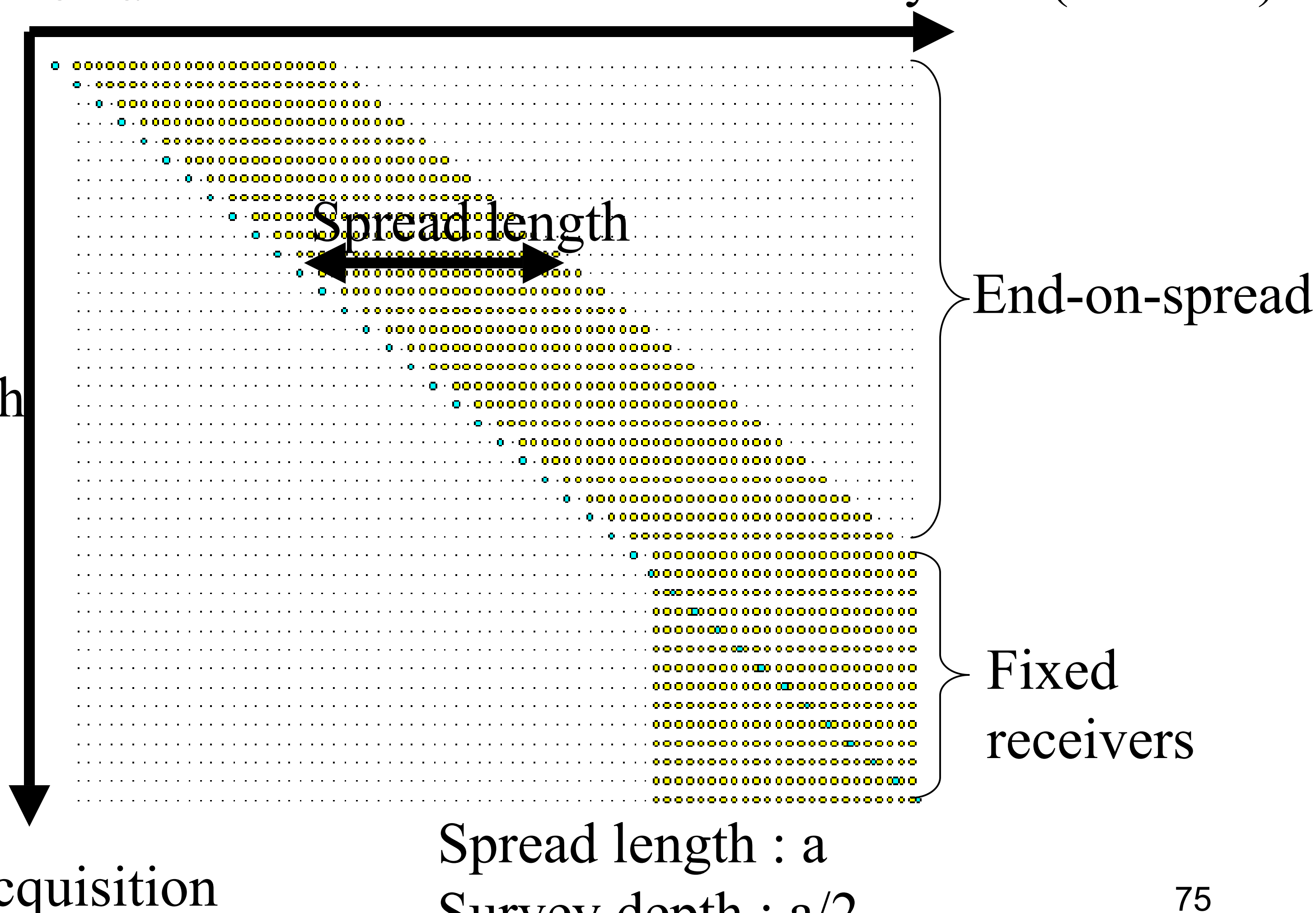


One spread length : a
Survey depth : a/4 to a/2

Source and receiver configuration for two-dimensional survey

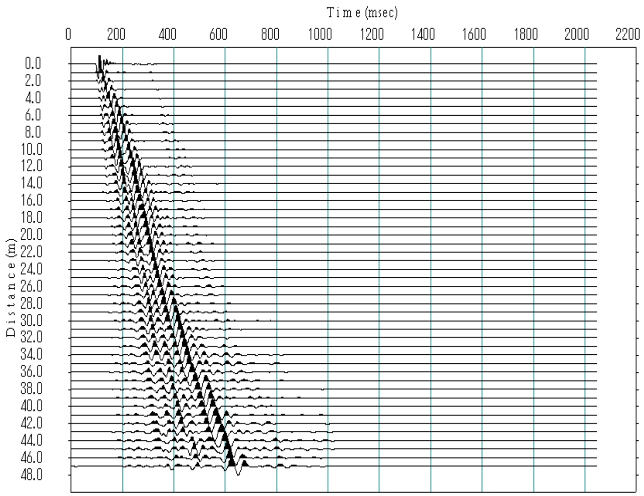
C) End-on-spread

Data acquisition pattern is similar to the ordinal 2D reflection. It can be performed with CDP switch.

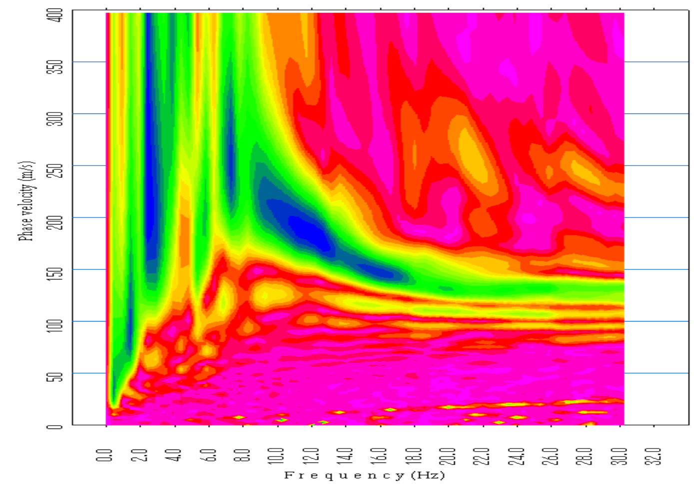


Spread length : a
Survey depth : a/2

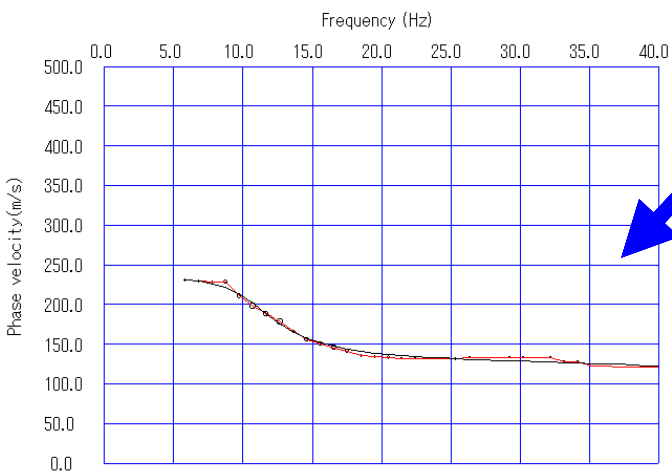
Dispersion curve and its analysis

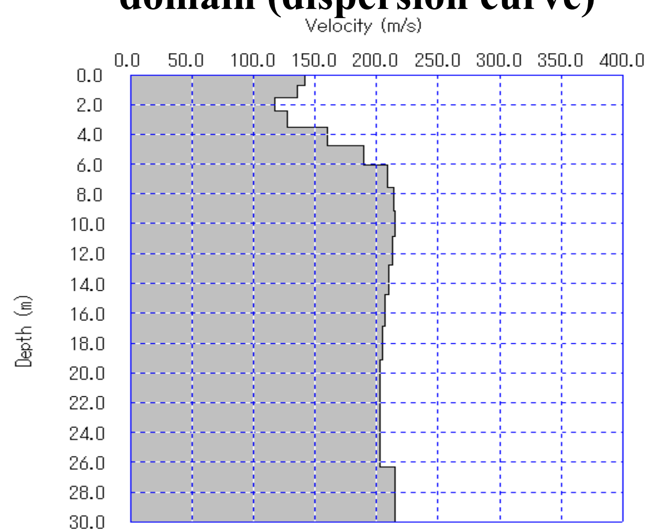


Common shot gather



Phase-velocity image in frequency domain (dispersion curve)



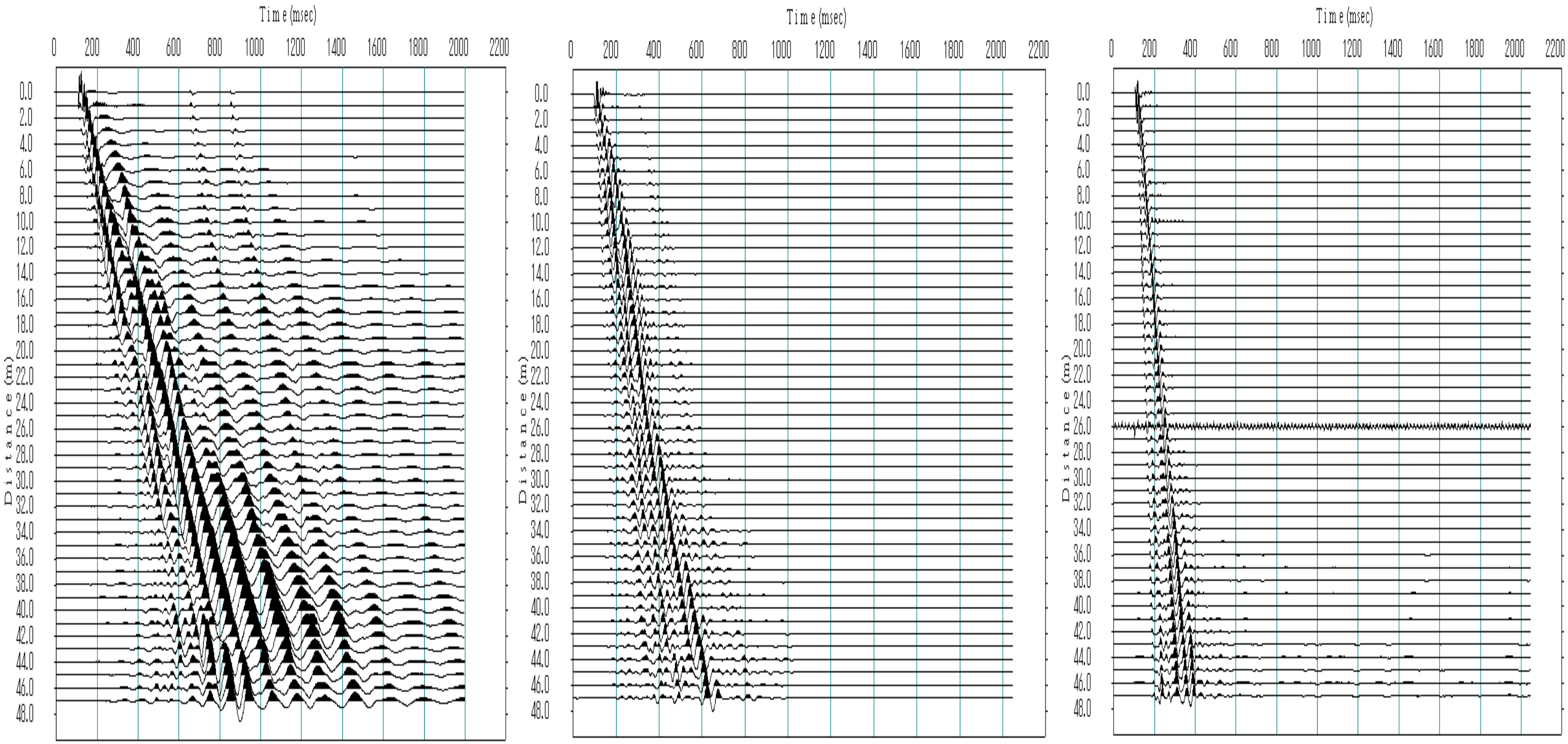


Dispersion curve inversion

1D S-wave velocity model

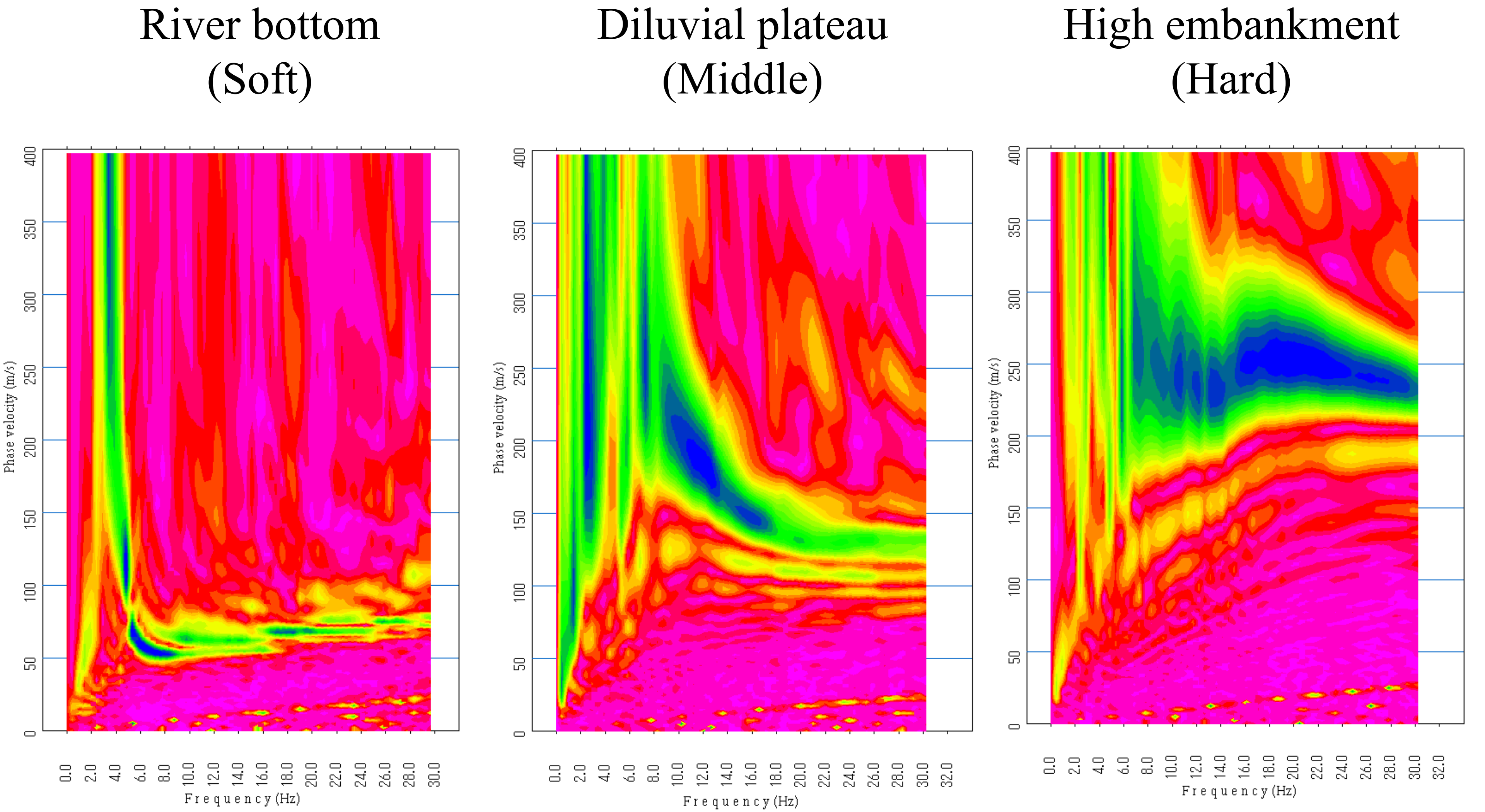
Comparison of the original waveforms

River bottom (Soft) | Diluvial plateau (Middle) | High embankment (Hard)



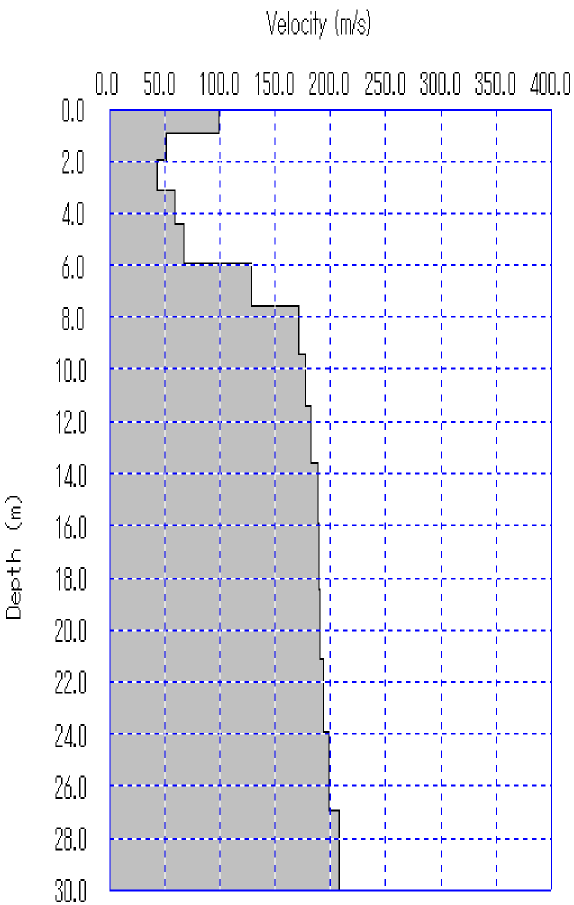
Source : Sledge hammer, Geophone : 4.5Hz, Receivers : 1m spacing x 48 points

Comparison of the dispersion curves

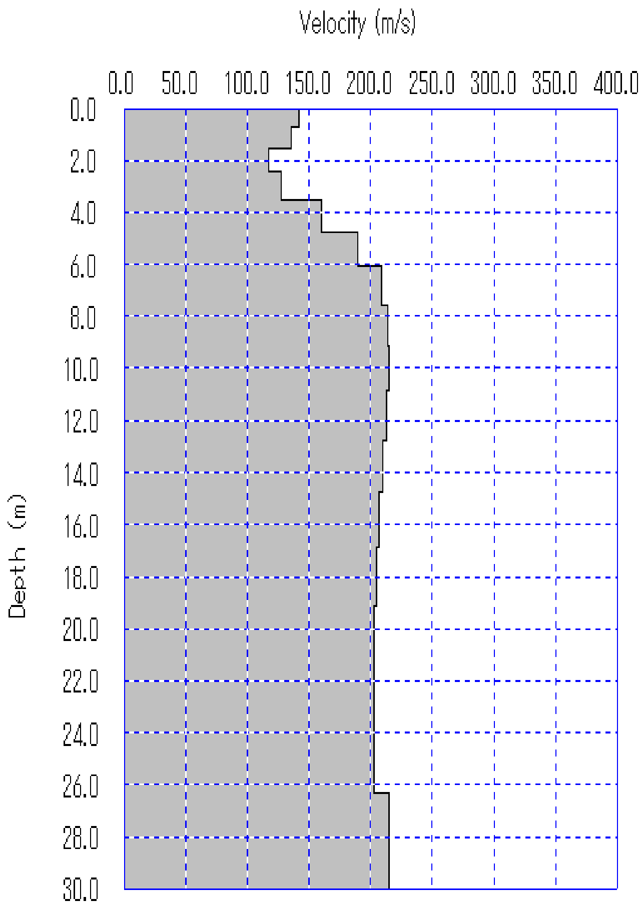


Comparison of the S-wave velocity structures

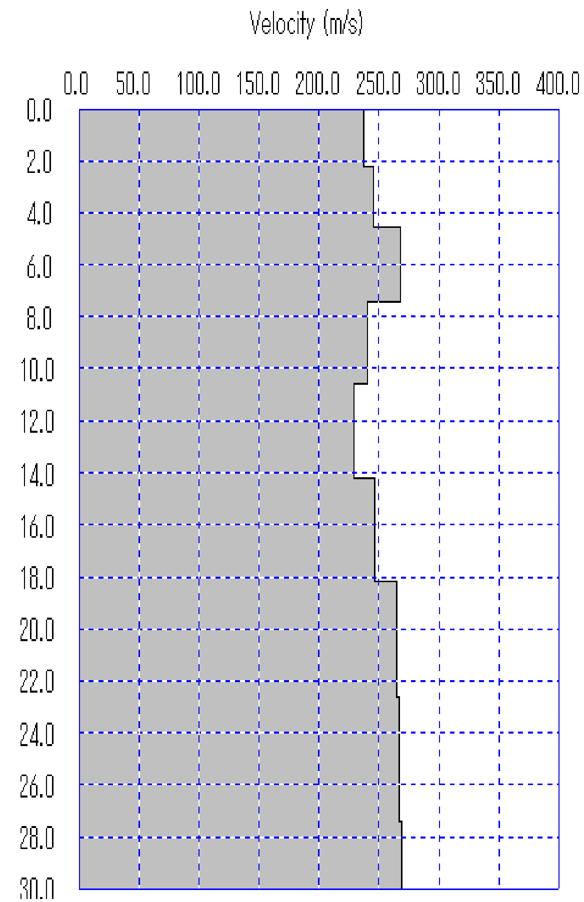
Riverbank
(Soft)



Diluvial plateau
(Middle)



High embankment
(Hard)



Passive Method

- Introduction to Surface Waves
- Fourier Transform
- Phase Velocity and Dispersion Curves
- Inversion
- Active Method
- Passive Method
- Application to Engineering Problems

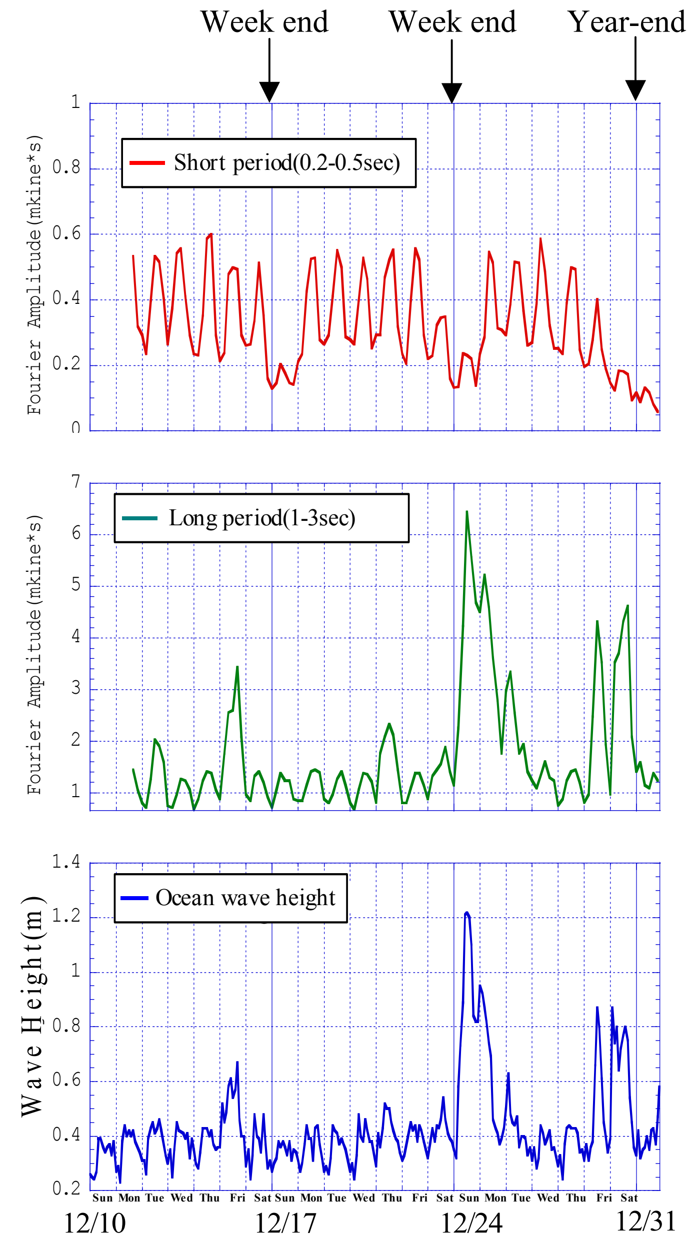
The soueces of microtremors

Short period microtremor
traffic noise and factory vibration

daily change

Long period microtremors
ocean waves and winds

High correlation with the ocean wave height



Data from SEO and YAMANAKA Lab. T.I.T.

Variation of Fourier amplitude of microtremors obaserved at KOBE area
and variation of sea wave hight of KOBE PORT

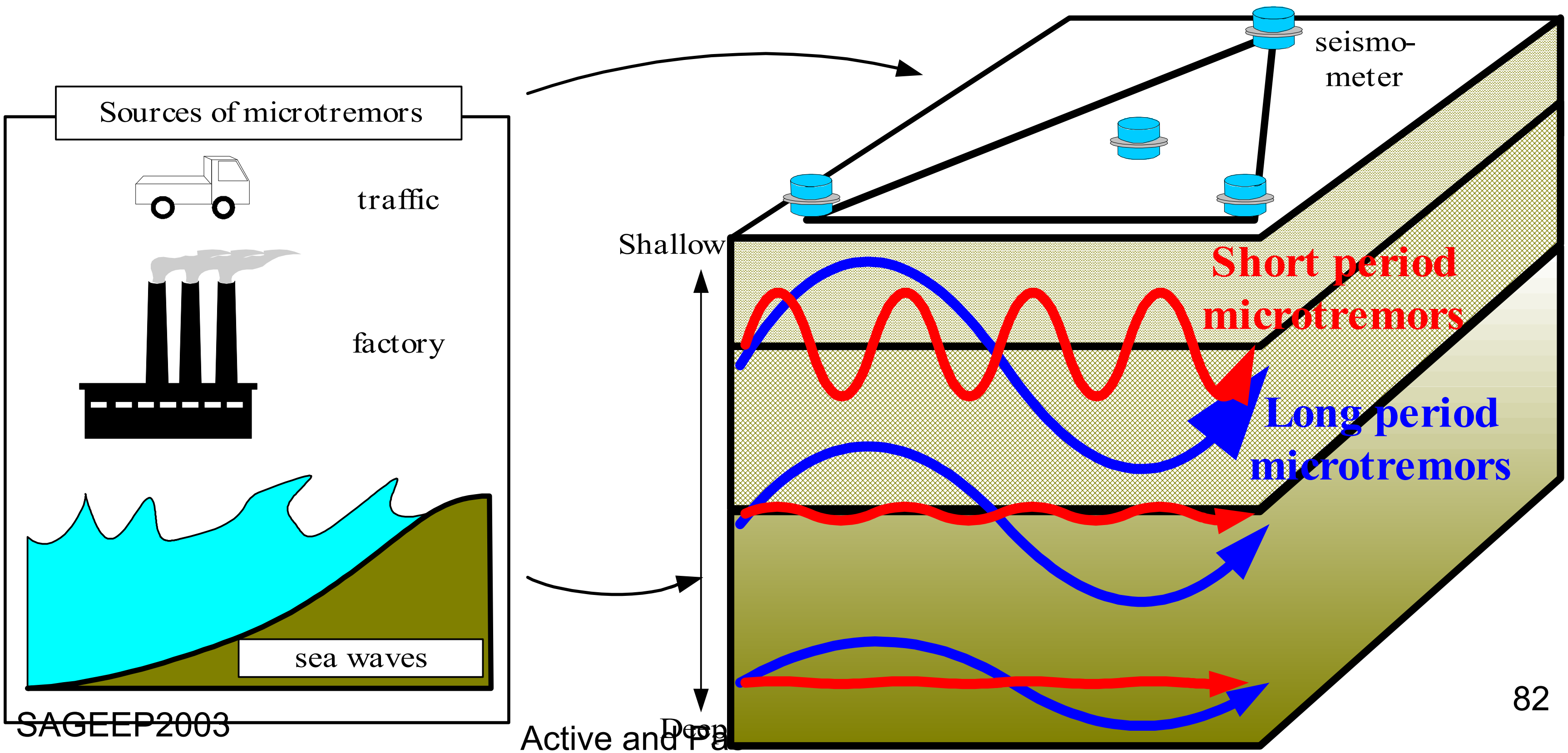
Characteristic of amplitude of surface wave

Energy concentrates near the ground surface.

Long period --> Deep depth
Short period --> Shallow depth

Surface wave dispersion

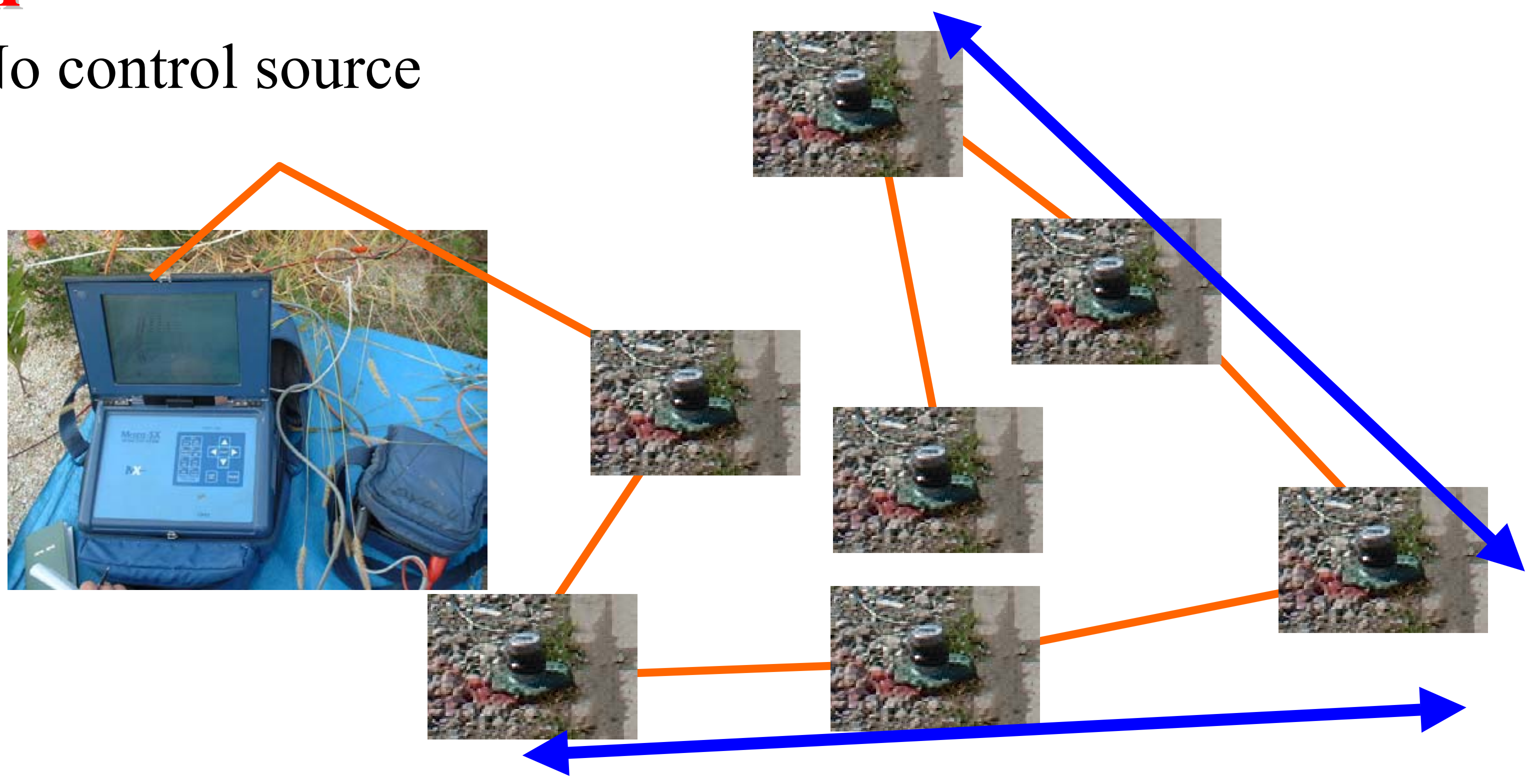
Long period --> High phase velocity
Short period --> Low phase velocity



Data acquisition of microtremors array measurements

Target □20□100m depth

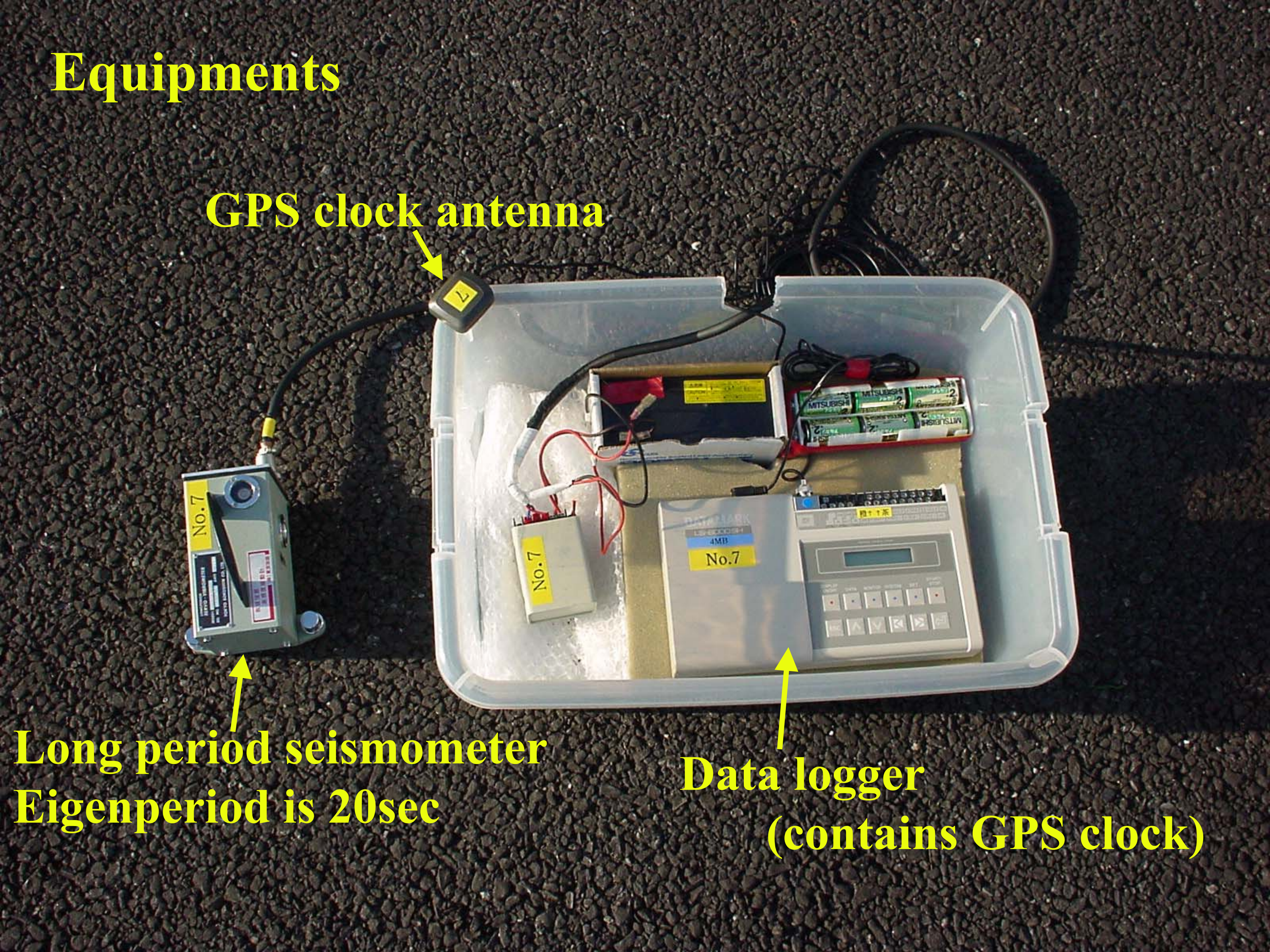
No control source



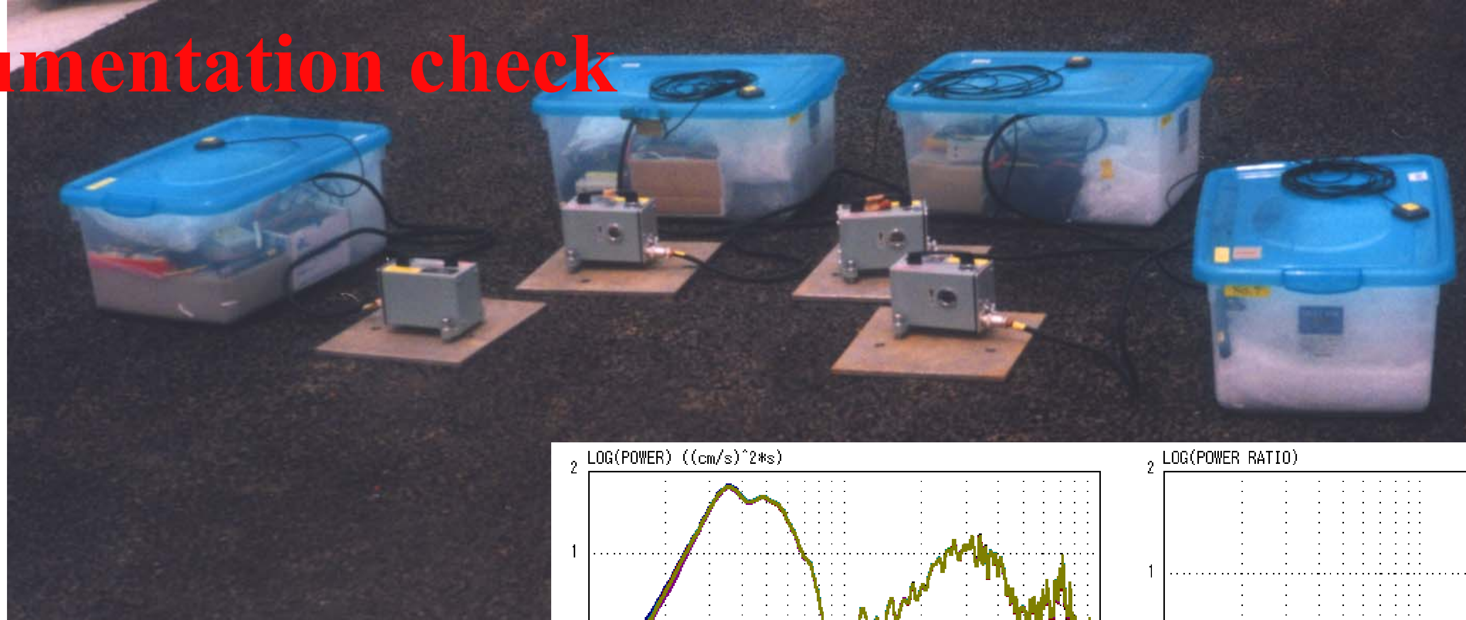
Receiver : 2Hz geophone

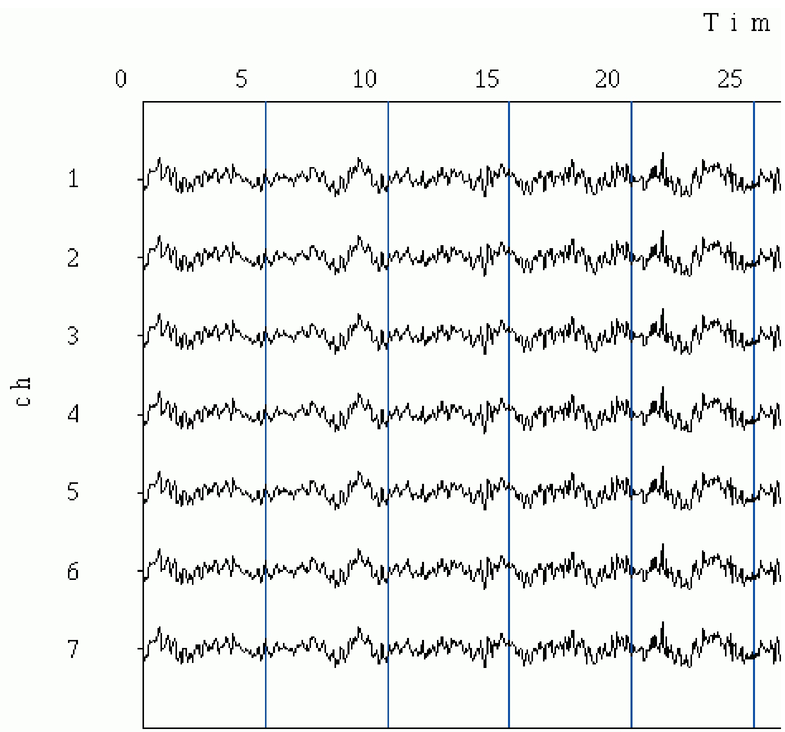
Long period seismometer

Array size□30□60m

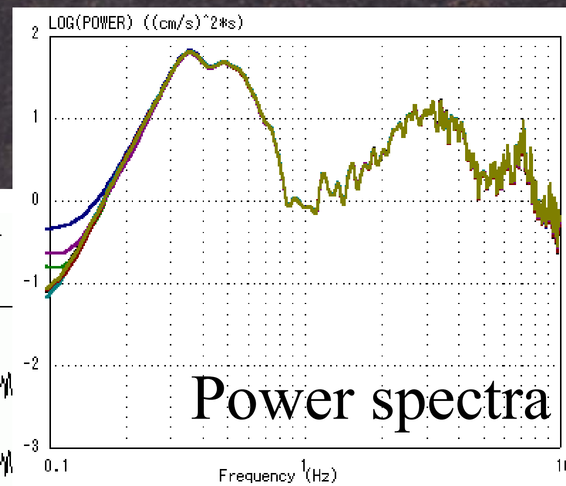

Equipments
GPS clock antenna
No.7
No.7
4MB
No.7
Long period seismometer
Eigenperiod is 20sec
Data logger
(contains GPS clock)

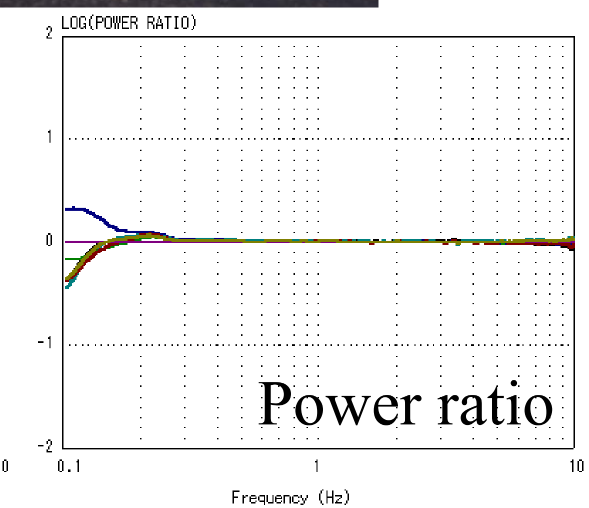
Instrumentation check

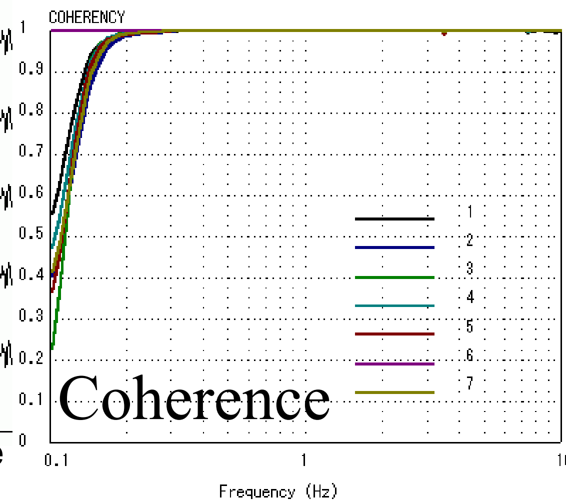


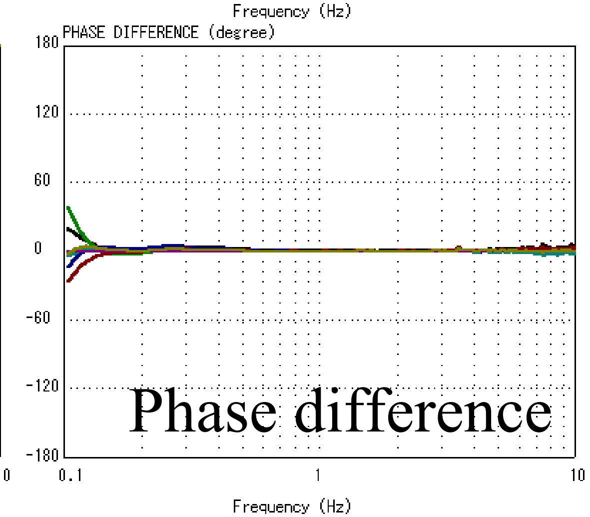


Wave forms

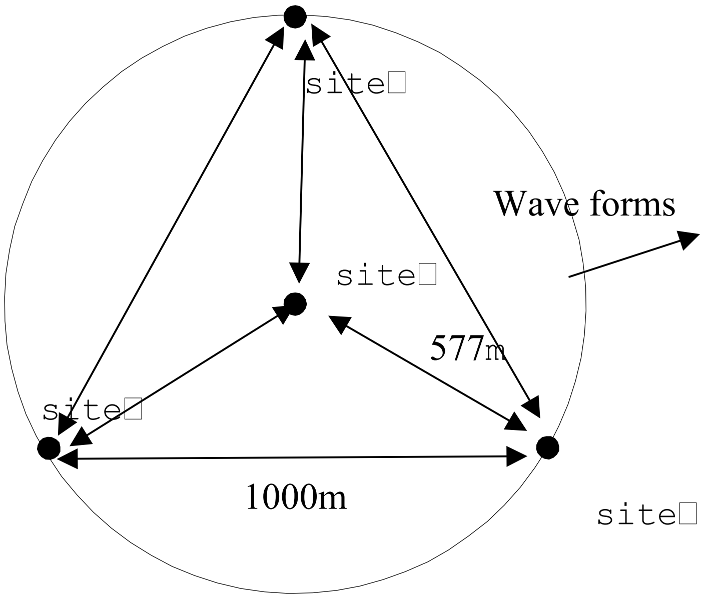


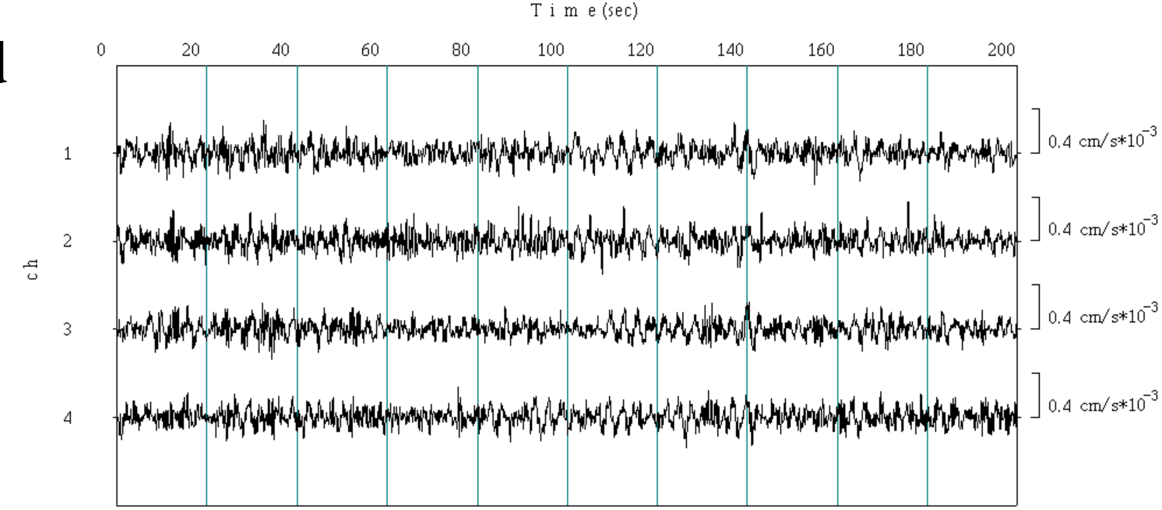






Analysis of SPAC method

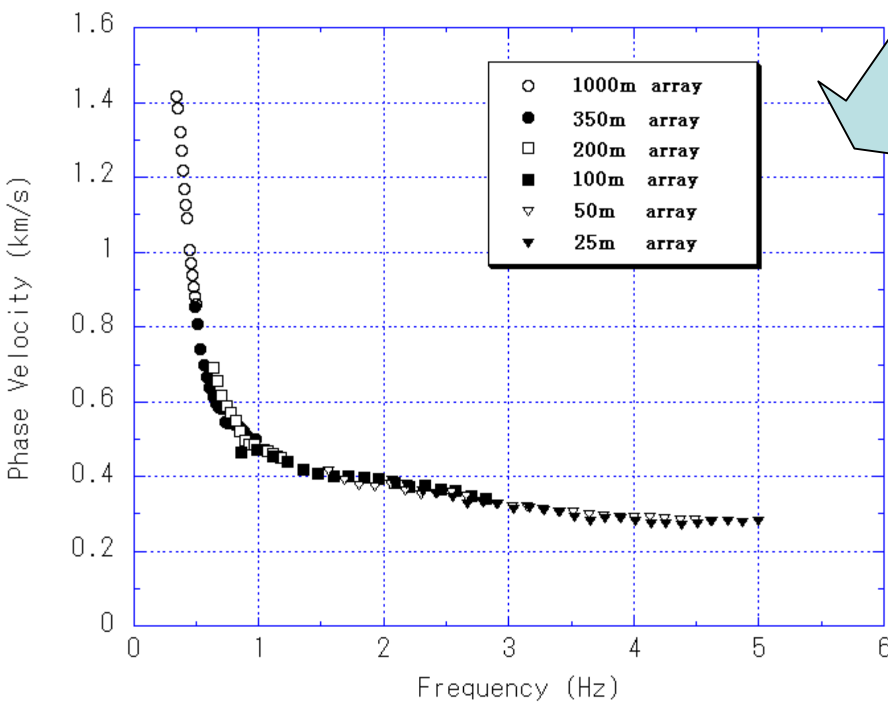




SPAC function$_{577m}$=(Coh$_{1-2}$+Coh$_{1-3}$+ Coh$_{1-4}$)/3

SPAC function$_{1000m}$=(Coh$_{2-3}$+Coh$_{2-4}$+Coh$_{3-4}$)/3

Coh$_{i-j}$ means coherence between sitei and sitej



SPAC function is expressed by Bessel function

SPAC function= $J_0(2\pi fr / c(f))$

Where , r is the distance between seismographs,c(f) is phase velocity of microtremors, J0 is Bessel function of first kind of order 0

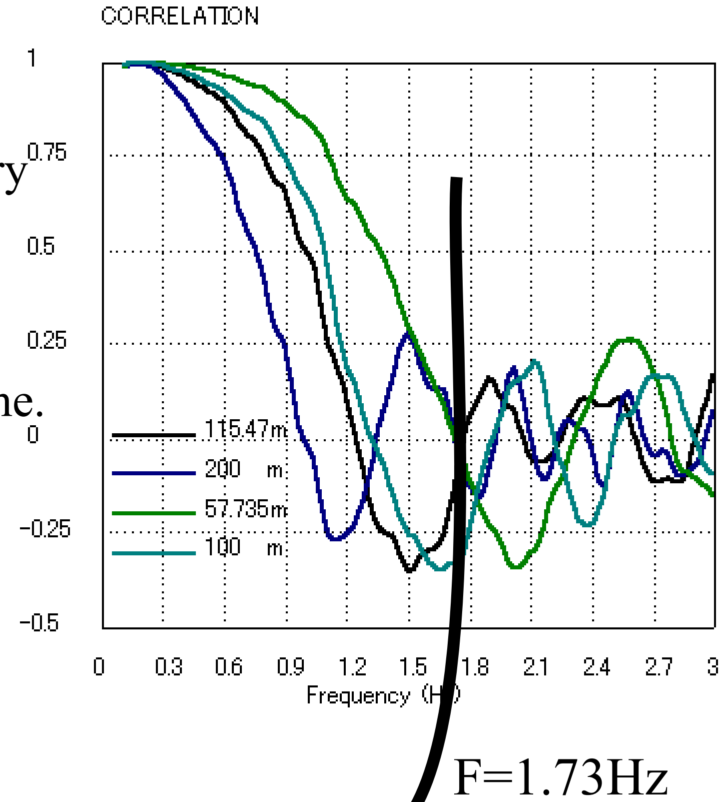
Analysis of SPAC method4

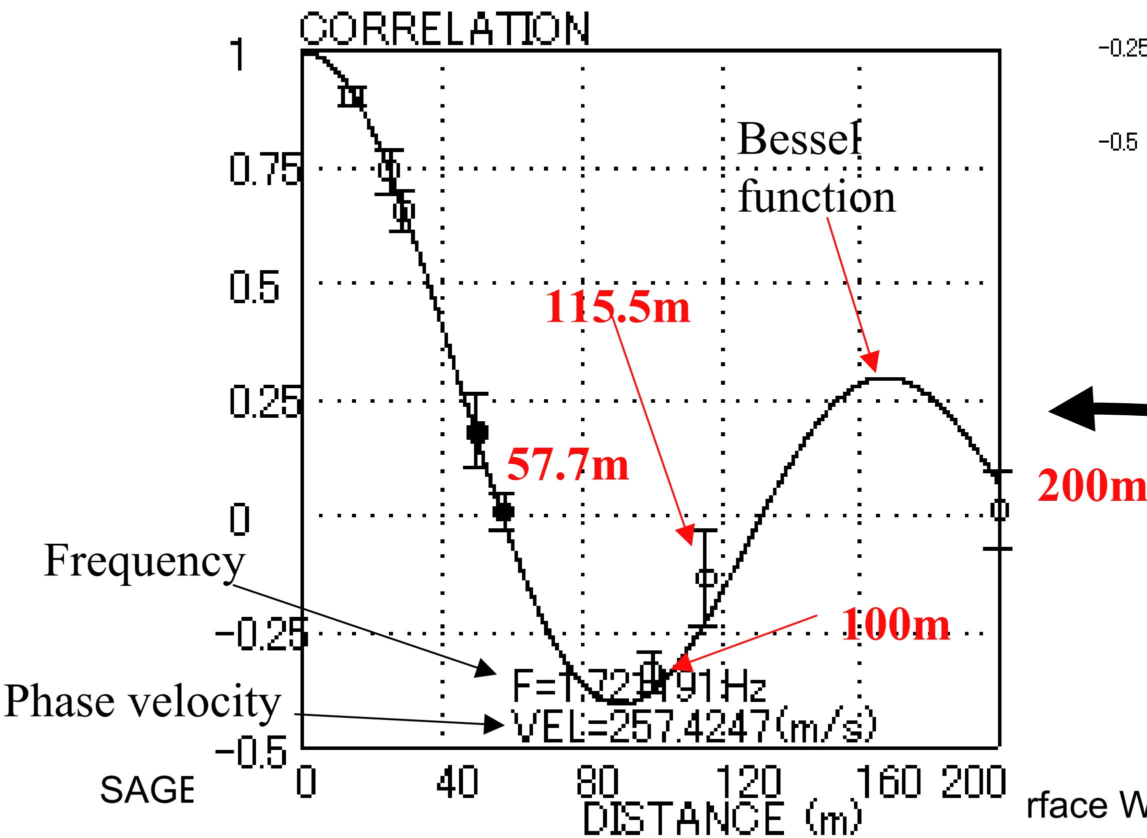
SPAC function= $J_0(2\pi fr / c(f))$

Phase velocity is calculated from SPAC function at every frequency.

By this method, various distance data can be unified.

The data is acquired on different day and at different time.

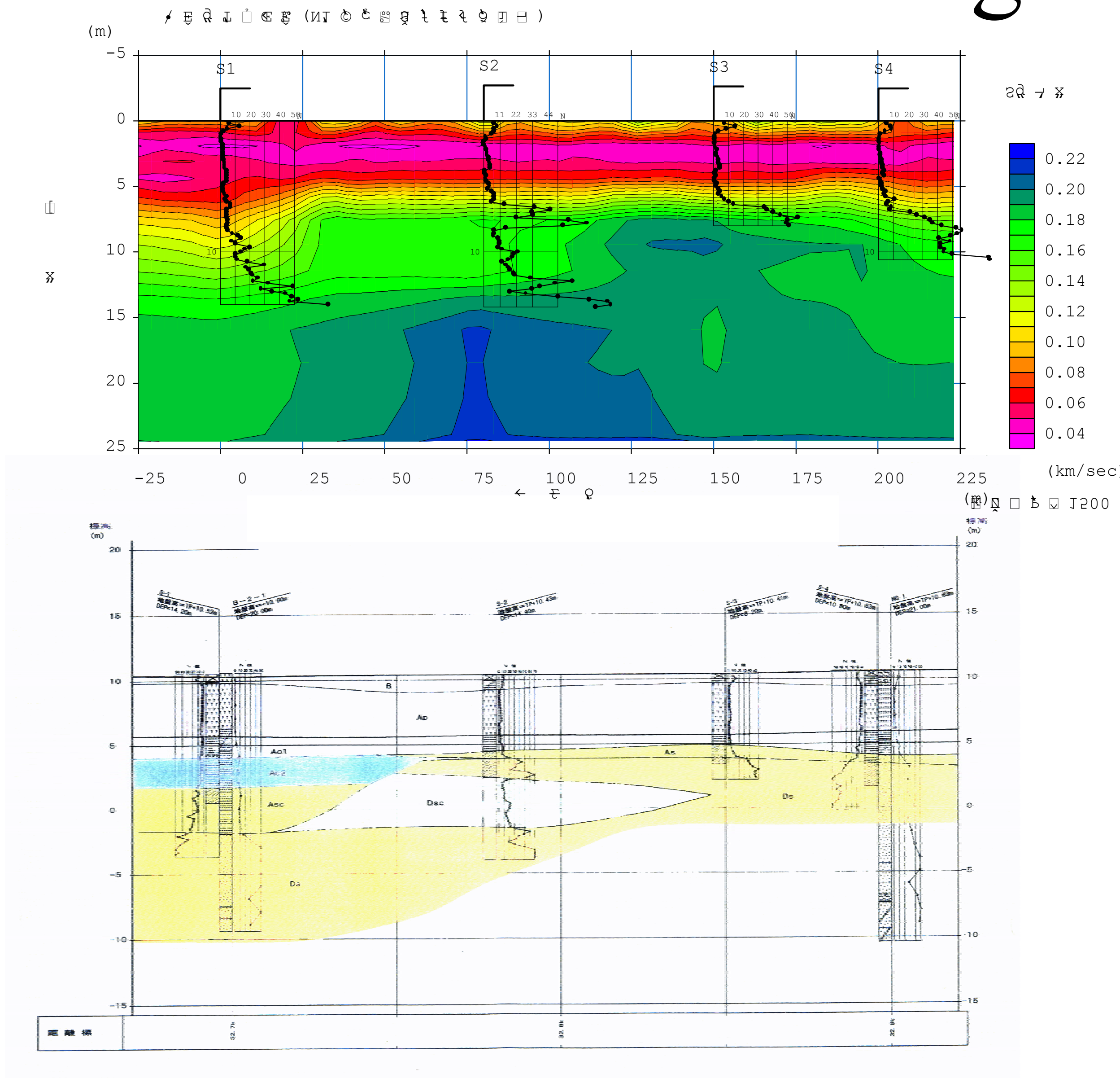




Application to Engineering Problems

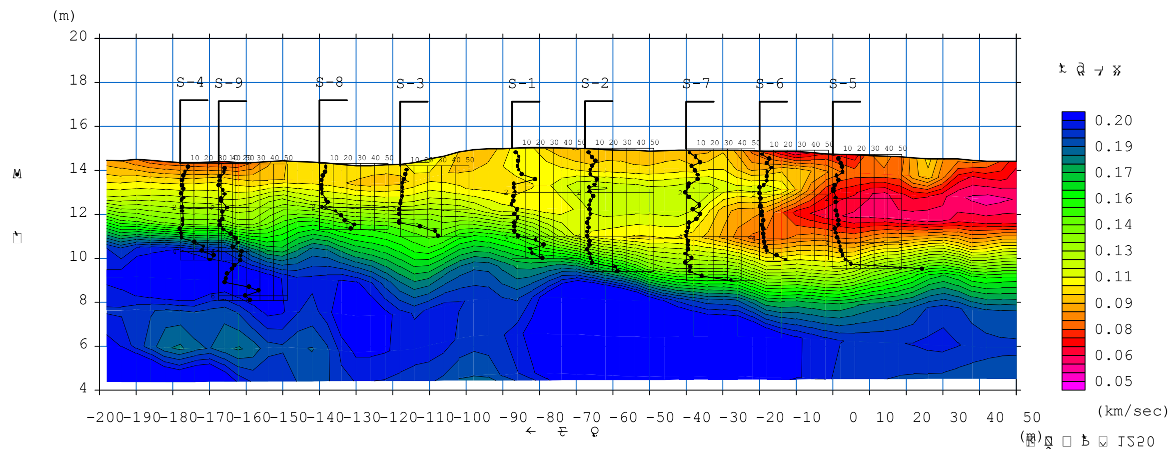
- Introduction to Surface Waves
- Fourier Transform, Phase Velocity and Dispersion
- Active Method
- Passive Method
- Application to Engineering Problems

Application of the surface-wave method to riverbank investigation

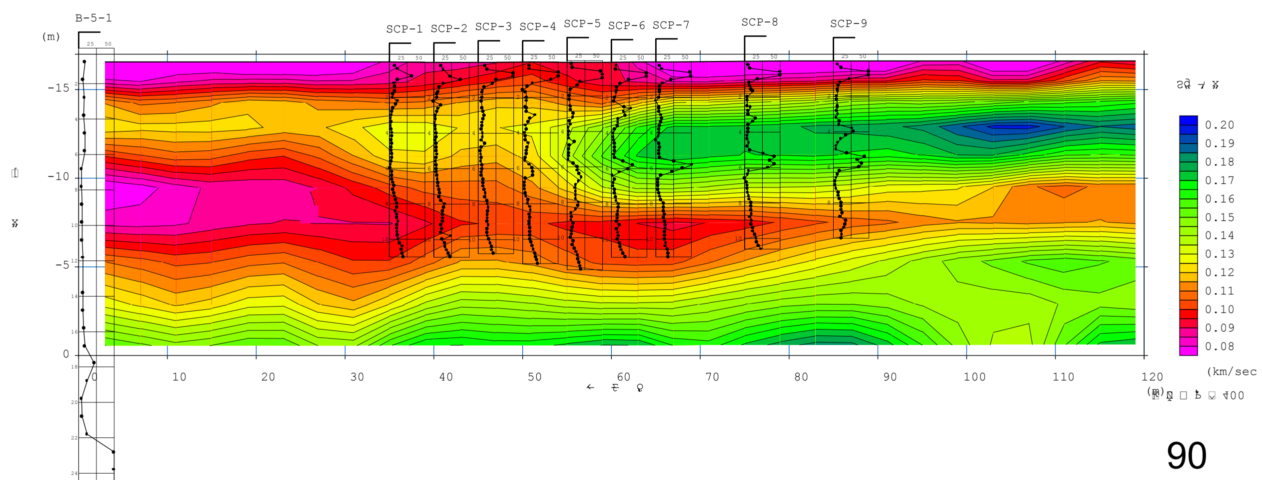


Application of the surface-wave method to riverbank investigation

Surveying old waterway



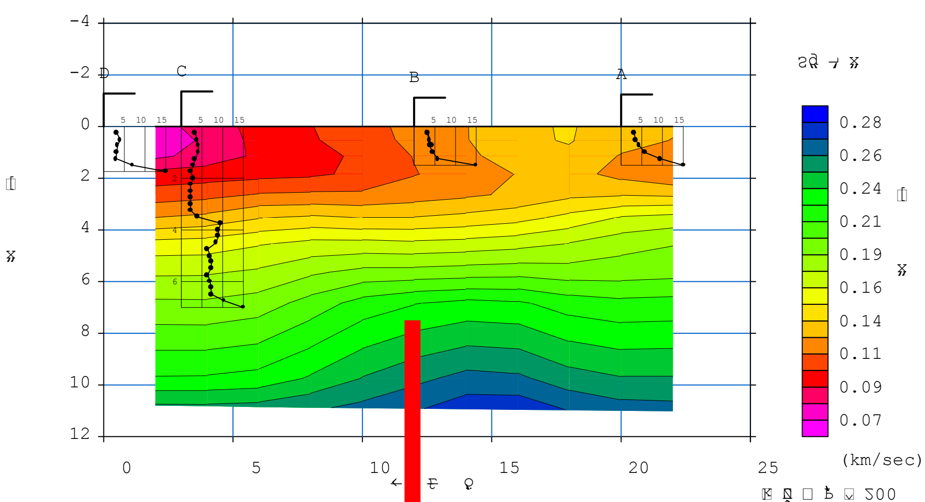
Understanding the area of soil improvement



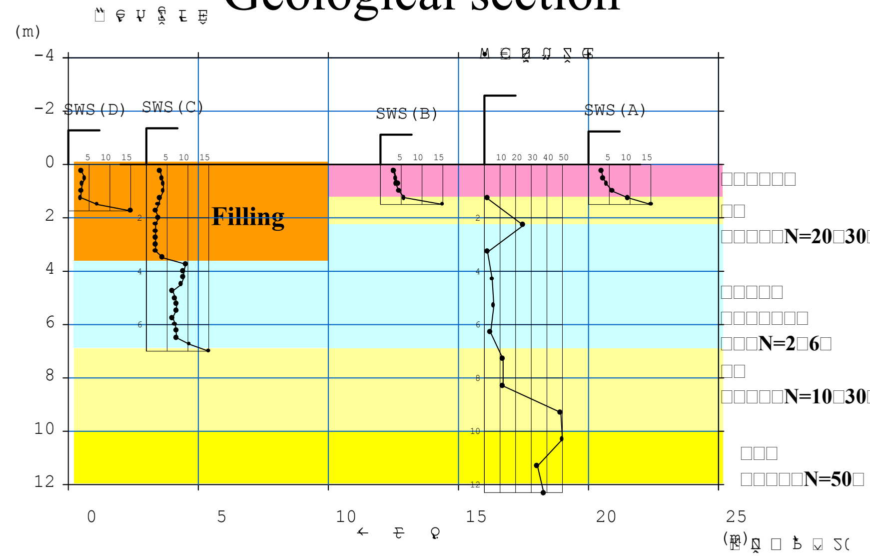
SAGEEP2003

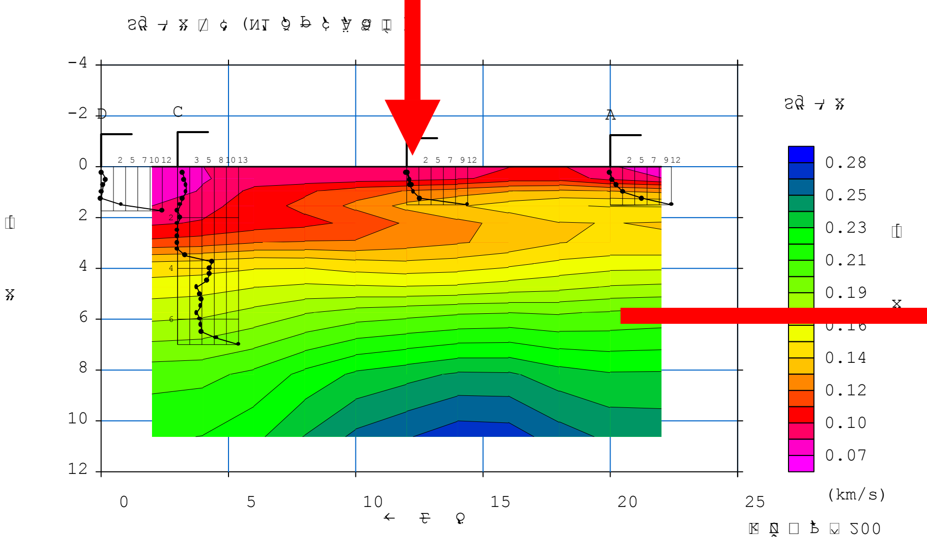
Application of the surface-wave method for understanding banking area

Result without N-value

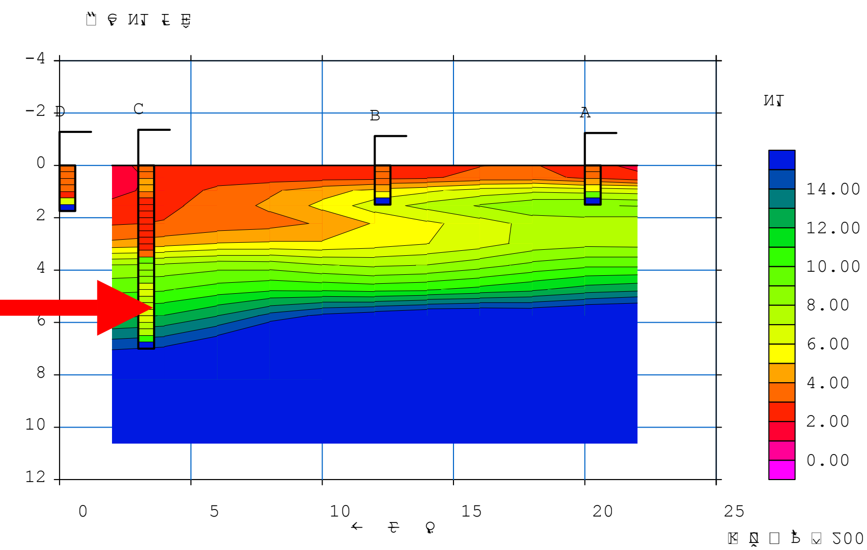


Geological section

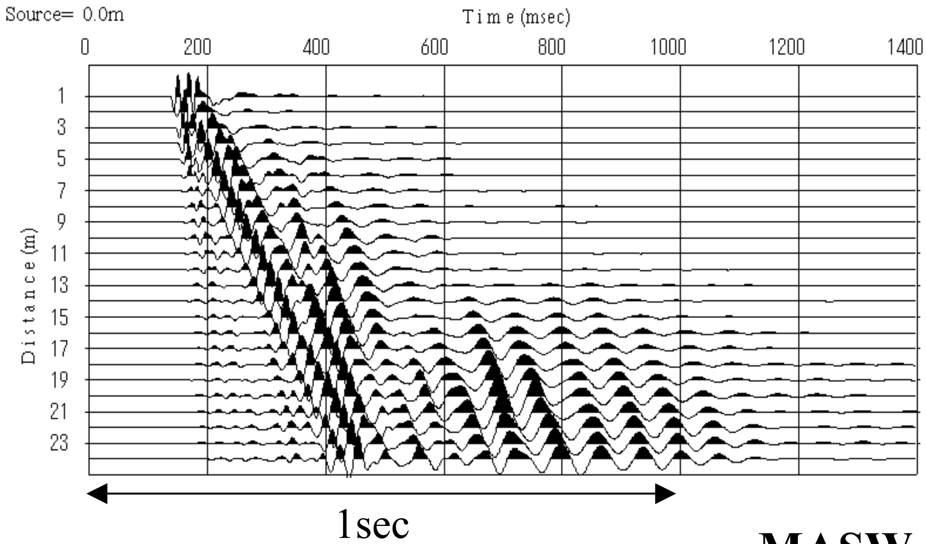


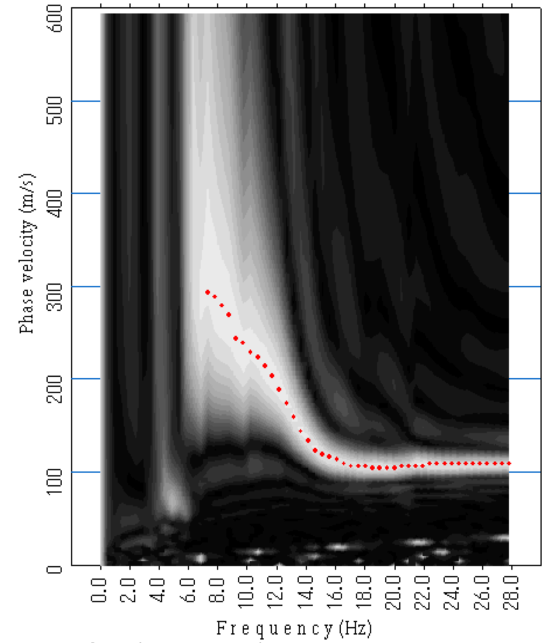


Result with N-value



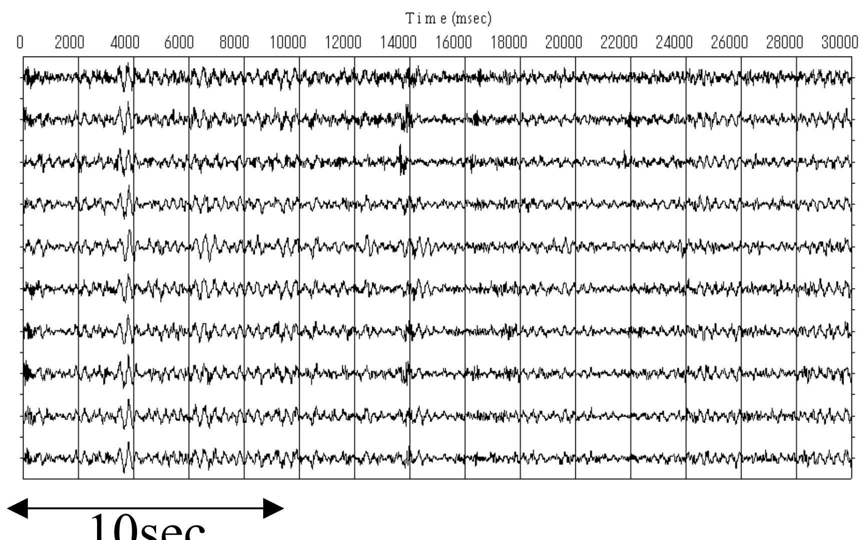
Estimated N-value structure

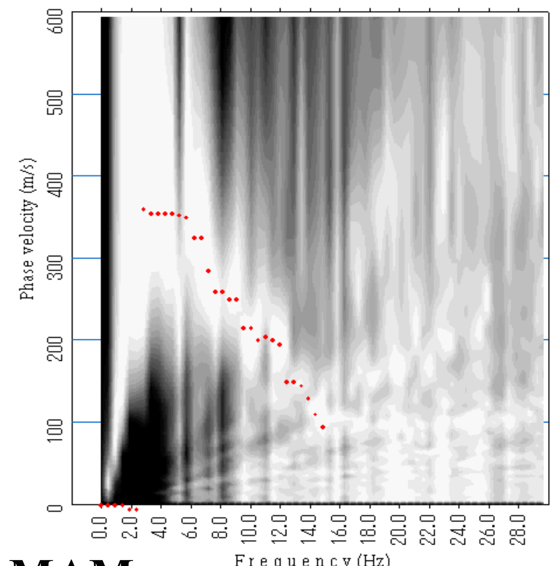




MASW

□□Wave forms□□□□□□□□□images of dispersion curves



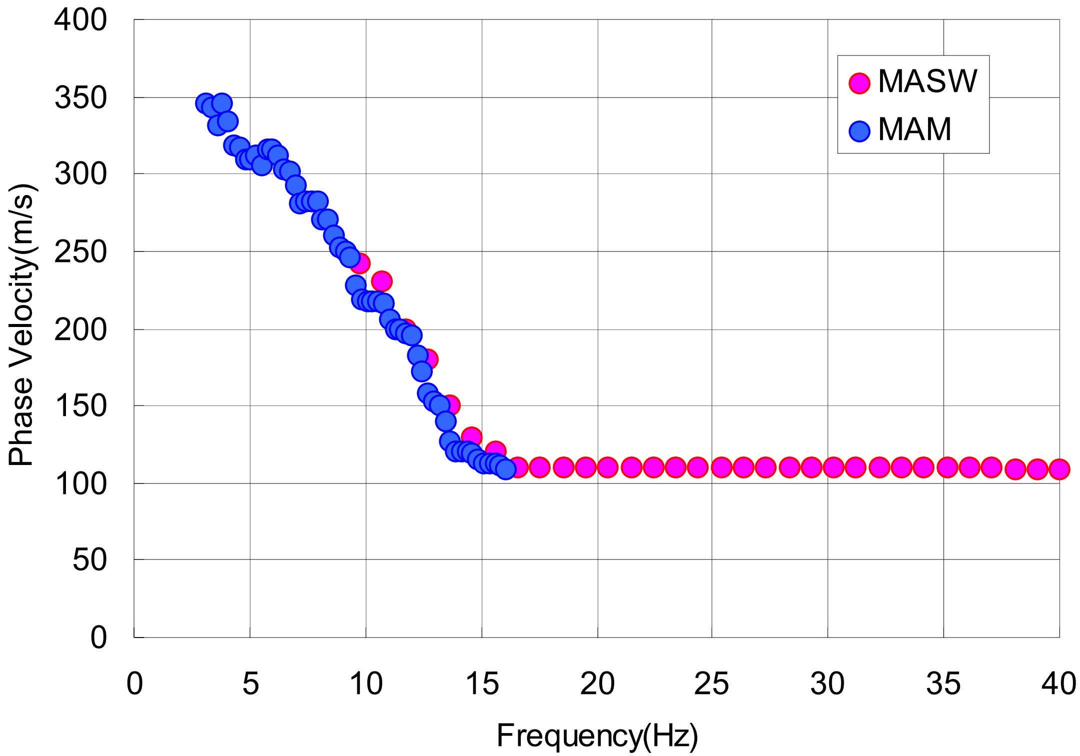


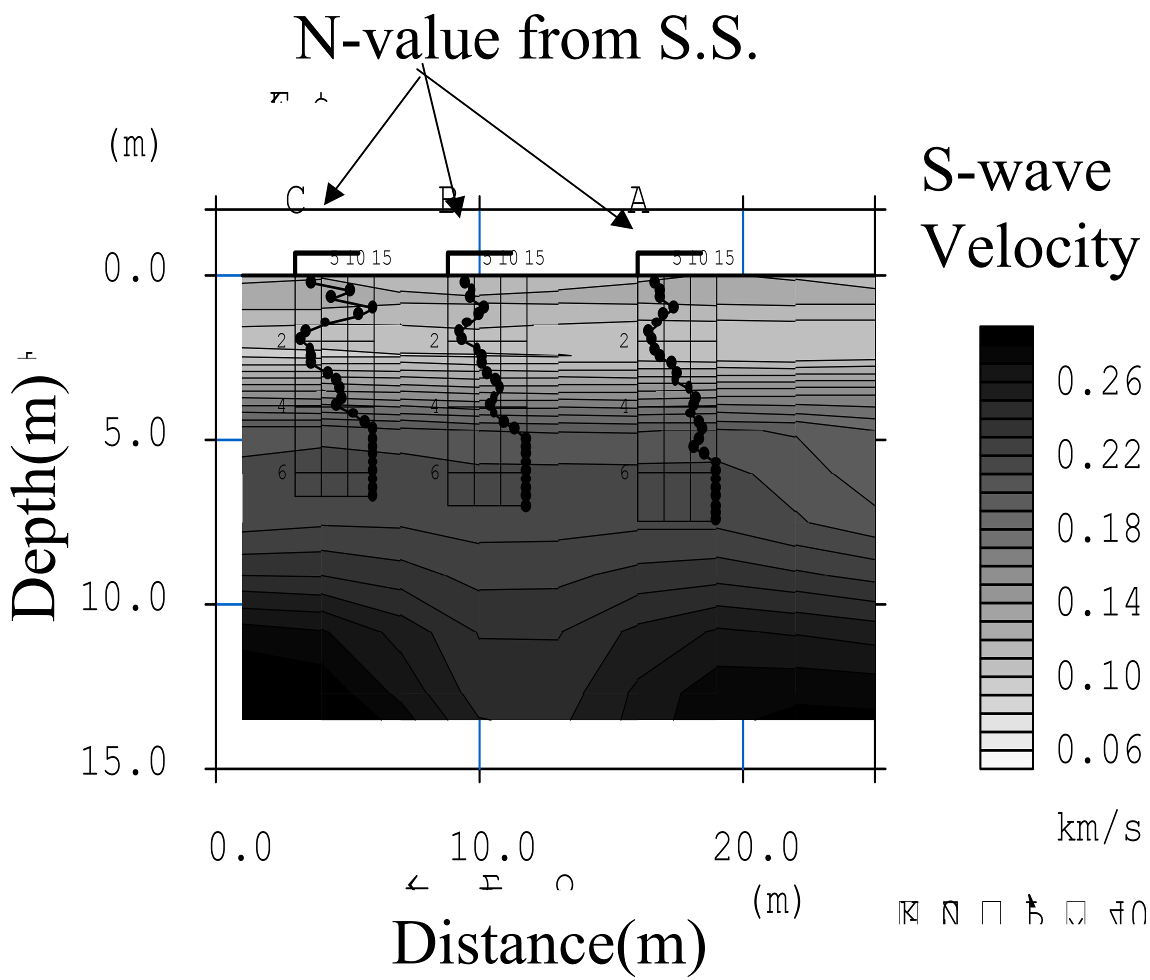
□□□□□□□□□□□□

MAM

□□Wave forms□□□□□□□□□images of dispersion curves

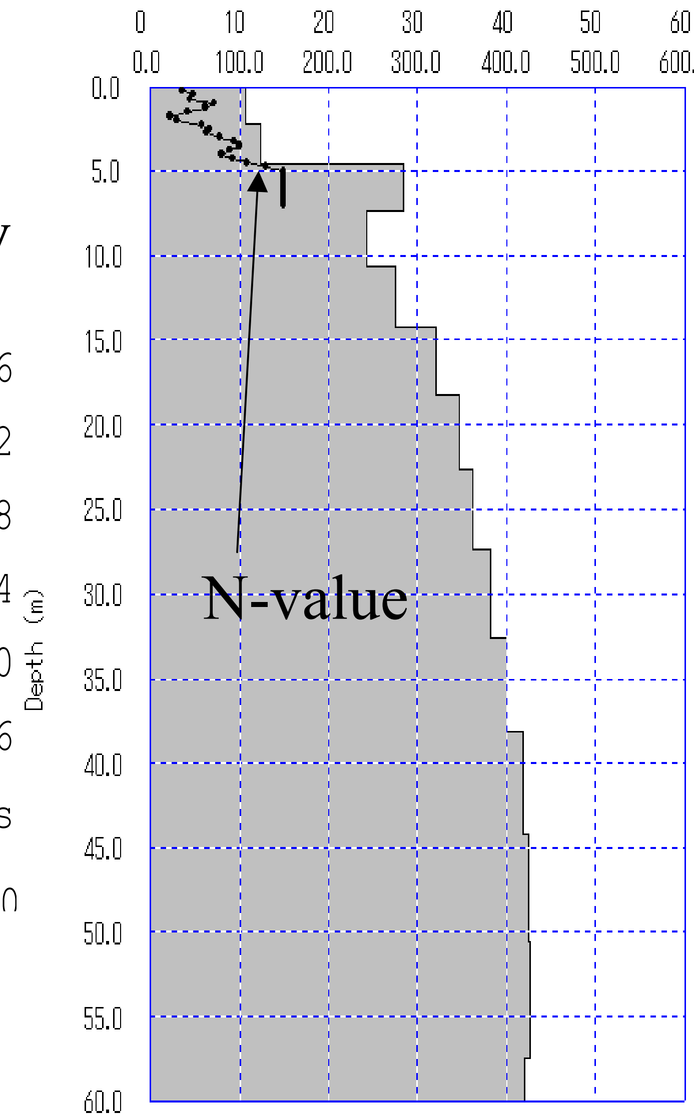
Dispersion curves from Active (MASW) and Passive (MAM) surface-wave methods





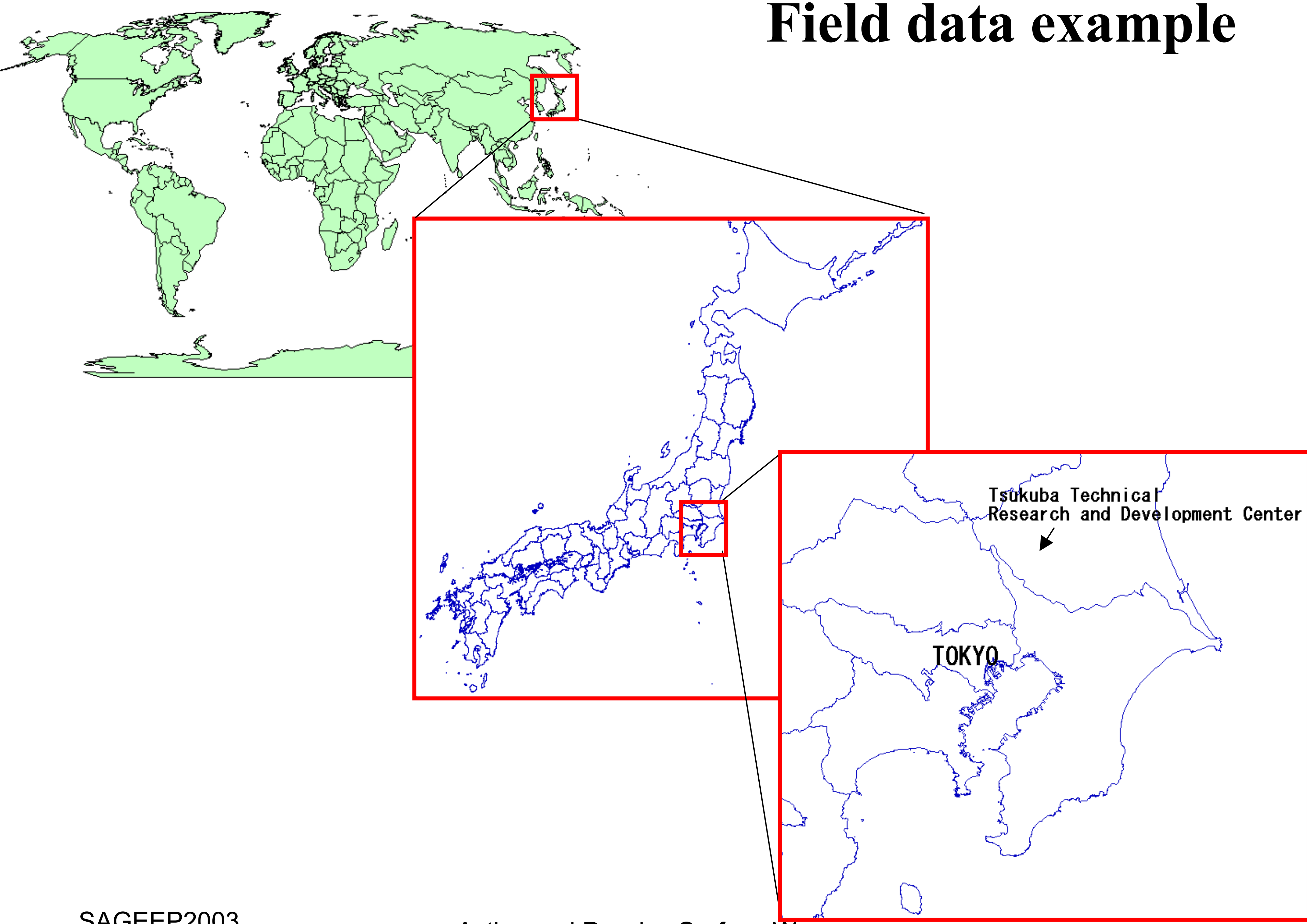
2D S-wave velocity structure by MASW

S-wave Velocity and N-value

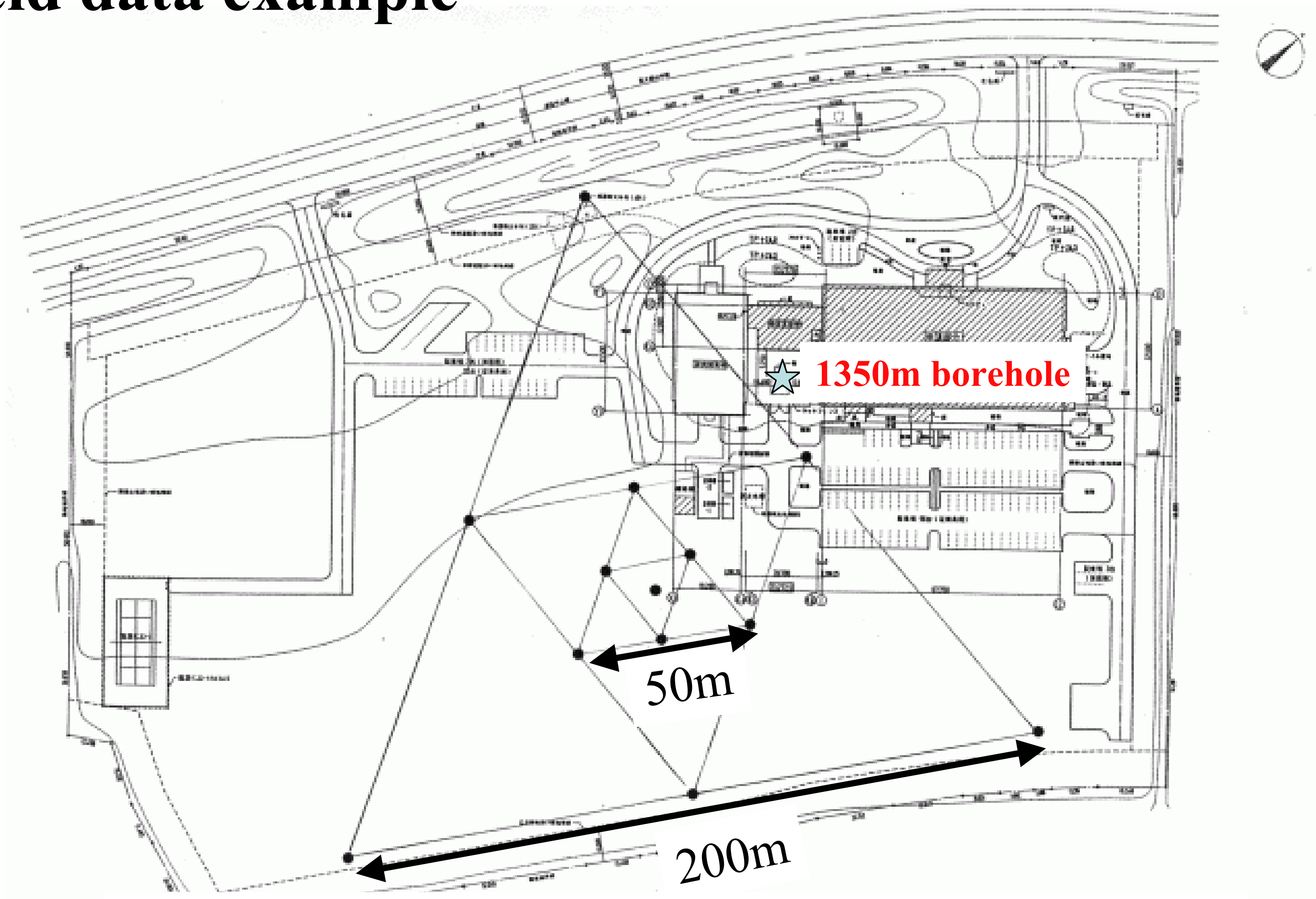


S-wave velocity structure obtained from MASW and MAM

Field data example

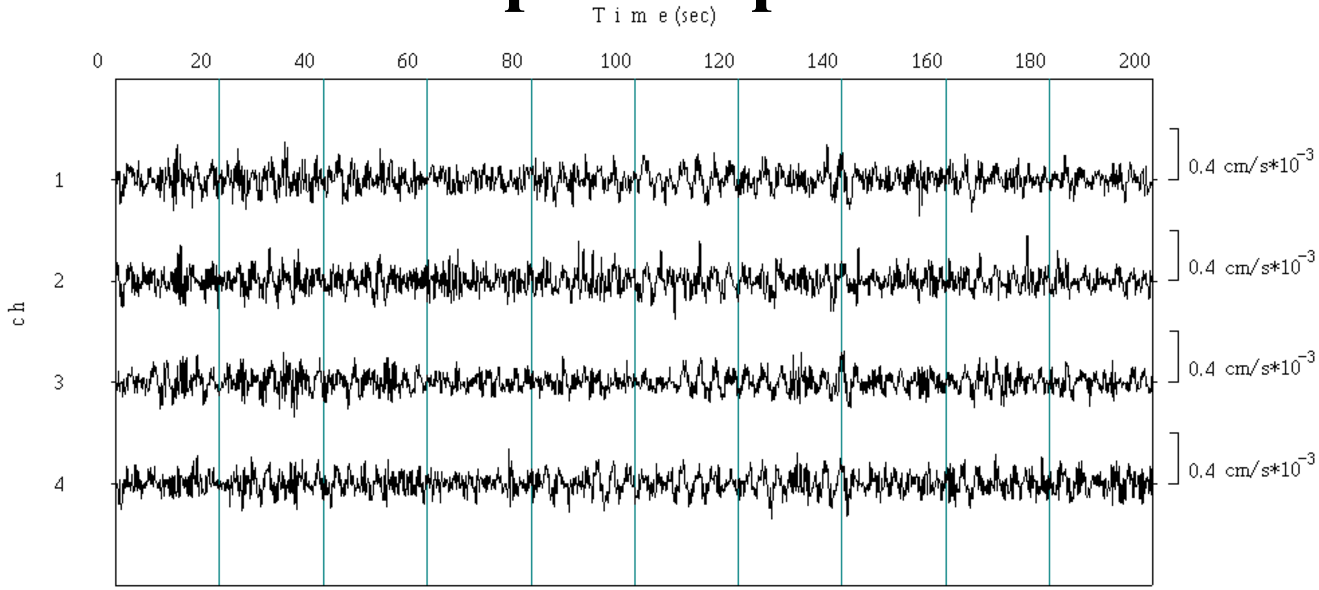


Field data example



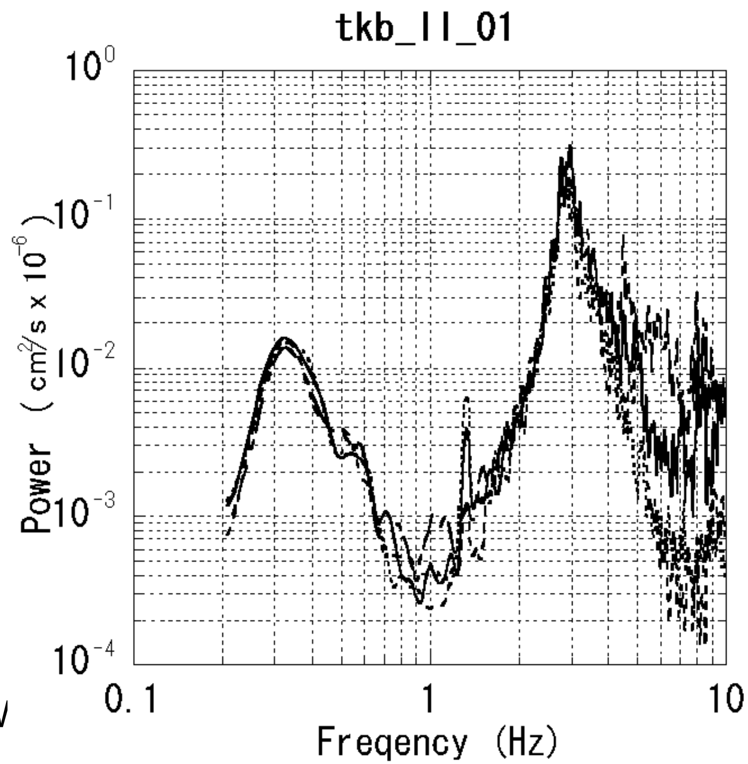
Observation sites in Tsukuba Technical R&D Center

Waveforms and power spectra



Example of waveforms at 1000m array

(2Hz high cut filtered)



Example of power spectra

1000m array data

S-wave velocity estimation

Genetic algorism

↓ **By GA process, it is easy to find many better models**

50 better models

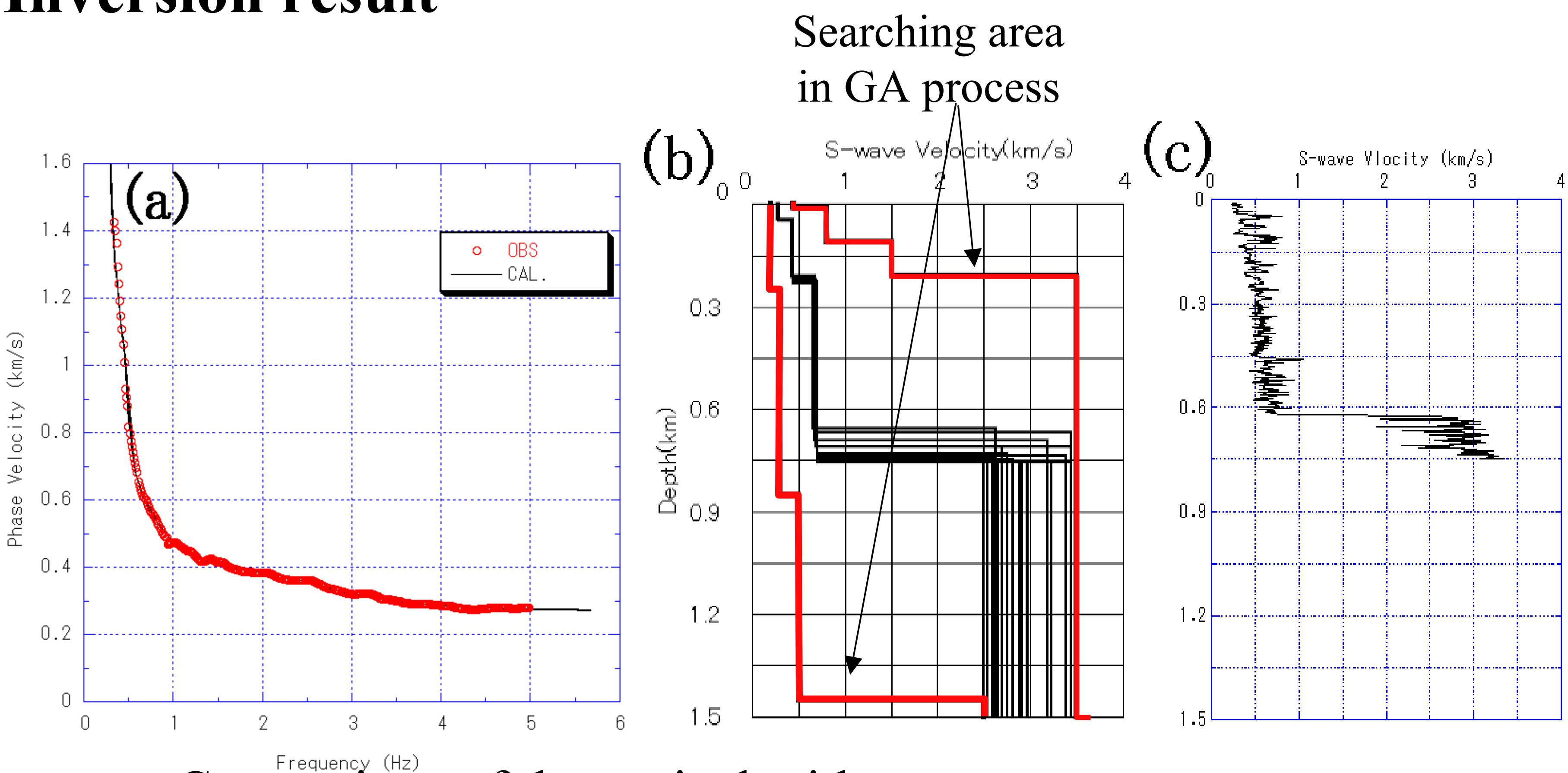
↓ **Non-linear least square method**

Final optimum velocity model

GA and least square inversion try to search which misfit is minimum

$$Misfit = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{V_o(f_i) - V_c(f_i)}{\sigma(f_i)} \right)^2$$

Inversion result



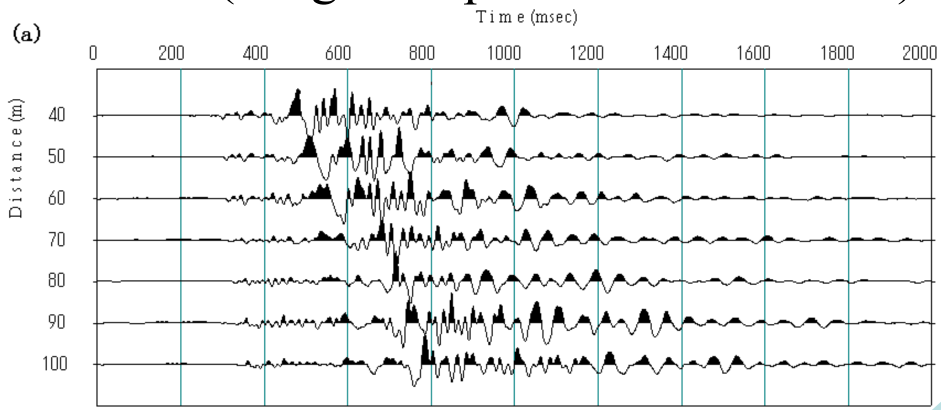
Comparison of theoretical with observed phase velocity

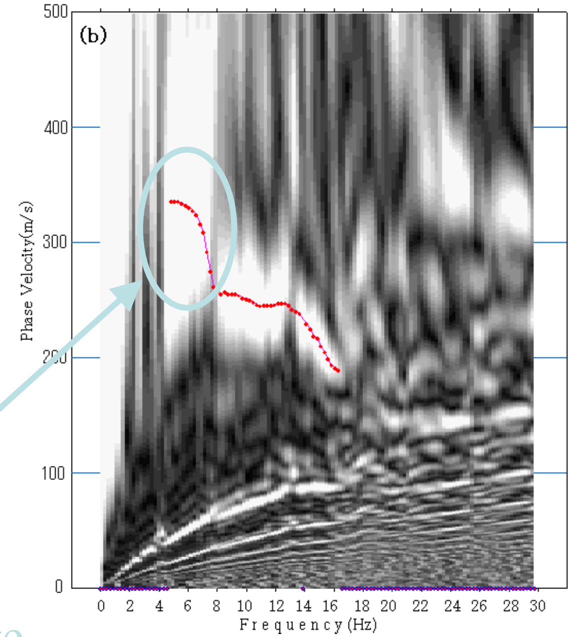
PS-logging data
Basement depth=625m

Black line:theoretical phase velocity curves
calculated by S-wave velocity structure (b)
(b) S-wave velocity structures misfit less than 0.056 are plotted.

Comparison with surface wave method data

Result of MASW (weight drop is used as a source)



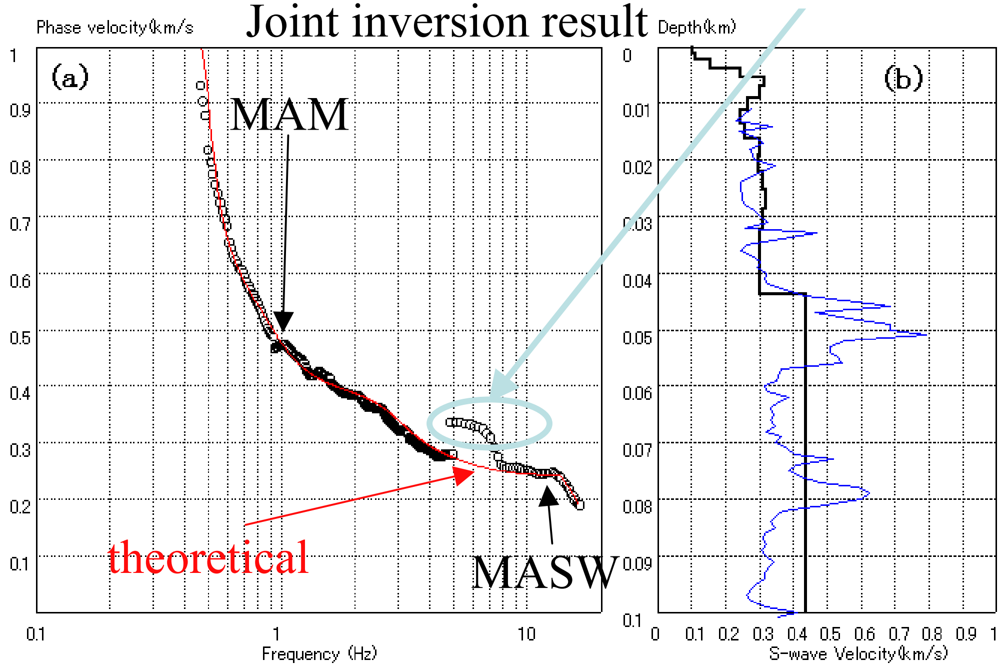


A weight drop was used as a source.
Seven VSE-12cc are used as a receiver.

Low resolution range

The image of dispersion curve.

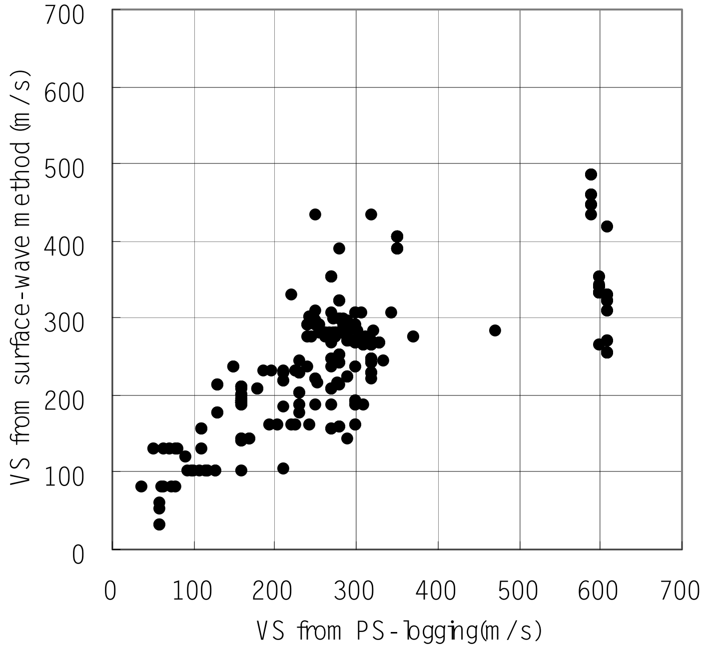
Joint inversion result



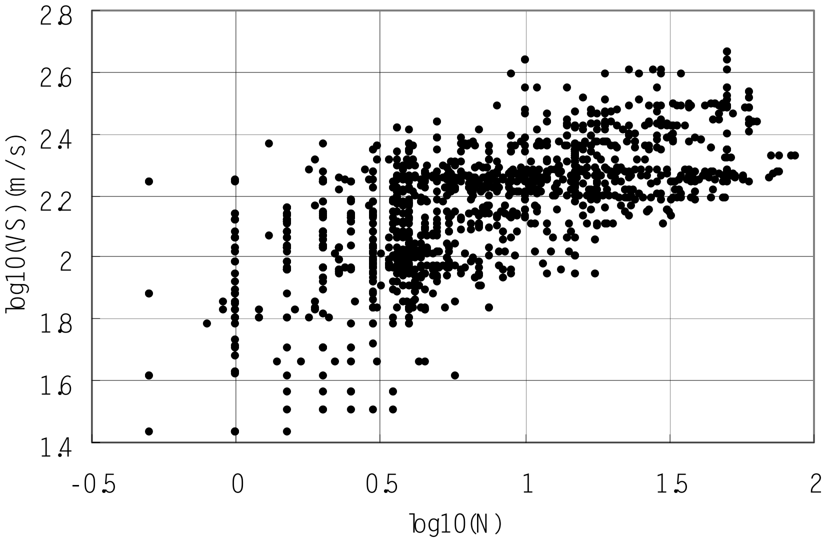
Using MAM and MASW, S-wave velocity structure from several meter to 700m can be estimated.

Comparison with PS-logging and N-value

Comparison with PS-logging



Comparison with N-value



Characteristics of S-wave velocity

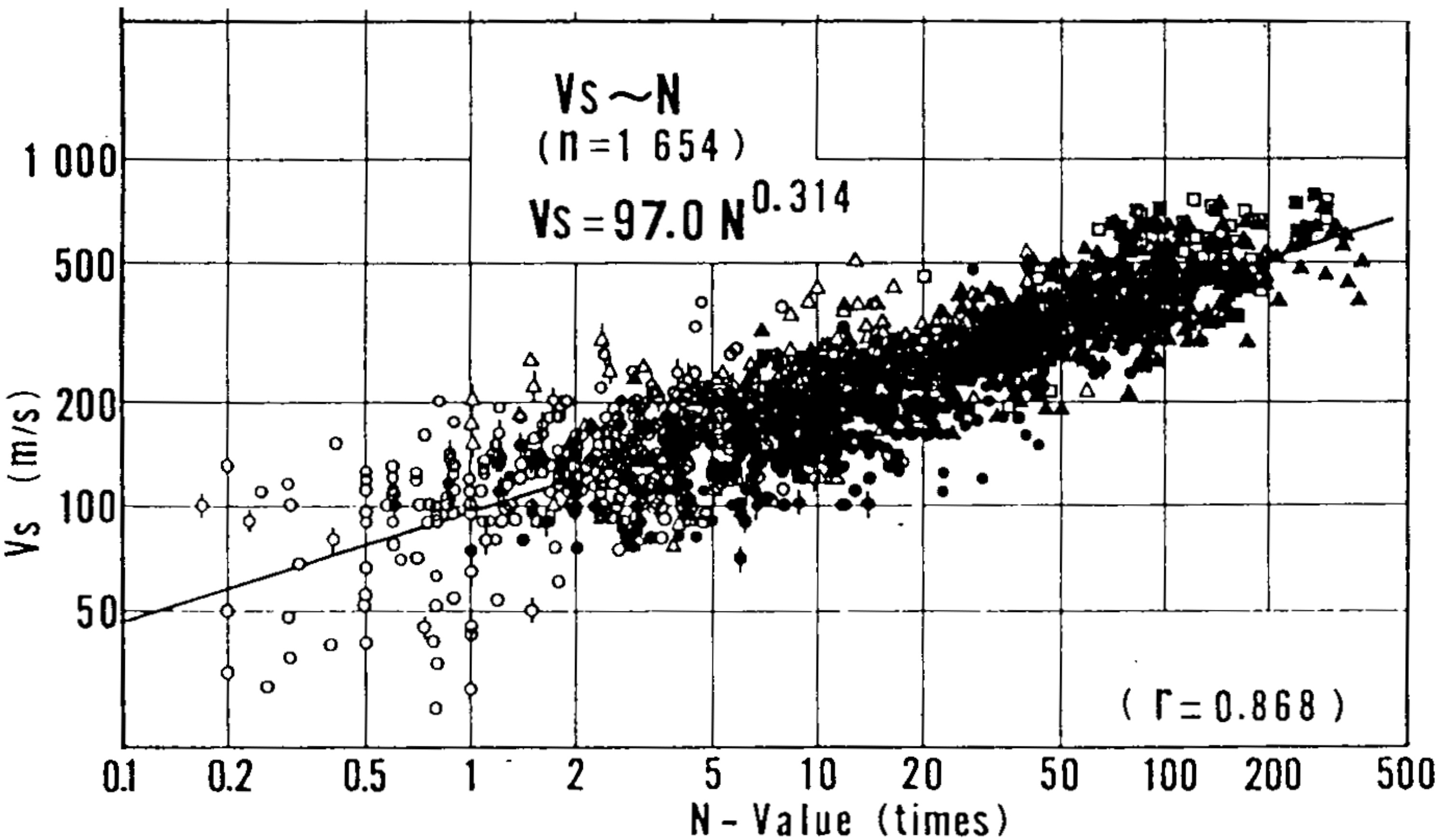
S-wave velocity (Vs) is defined by shear modulus (rigidity G) and density (ρ)

$$Vs = \sqrt{\frac{G}{\rho}} = \sqrt{\frac{\mathrm{E}}{2(1+\nu)\rho}}$$

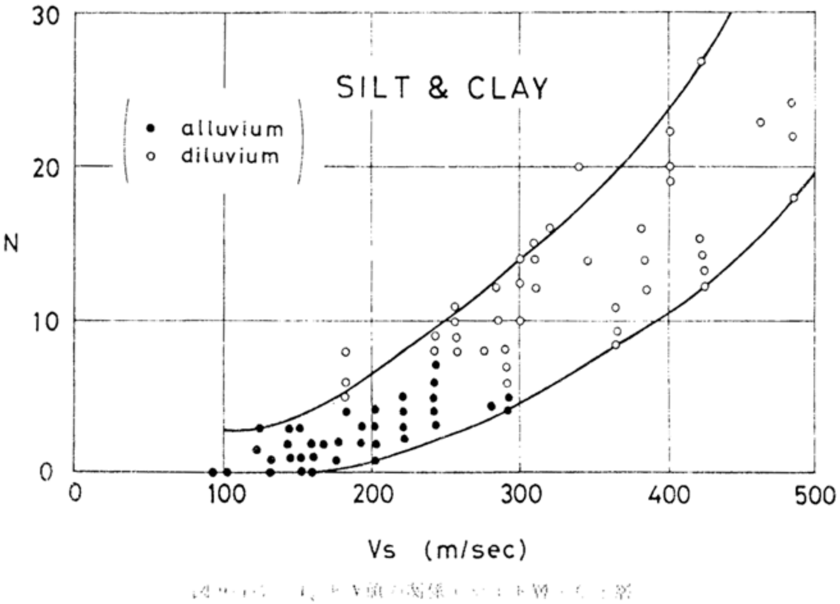
E : Young's modulus

ν : Possion's ratio

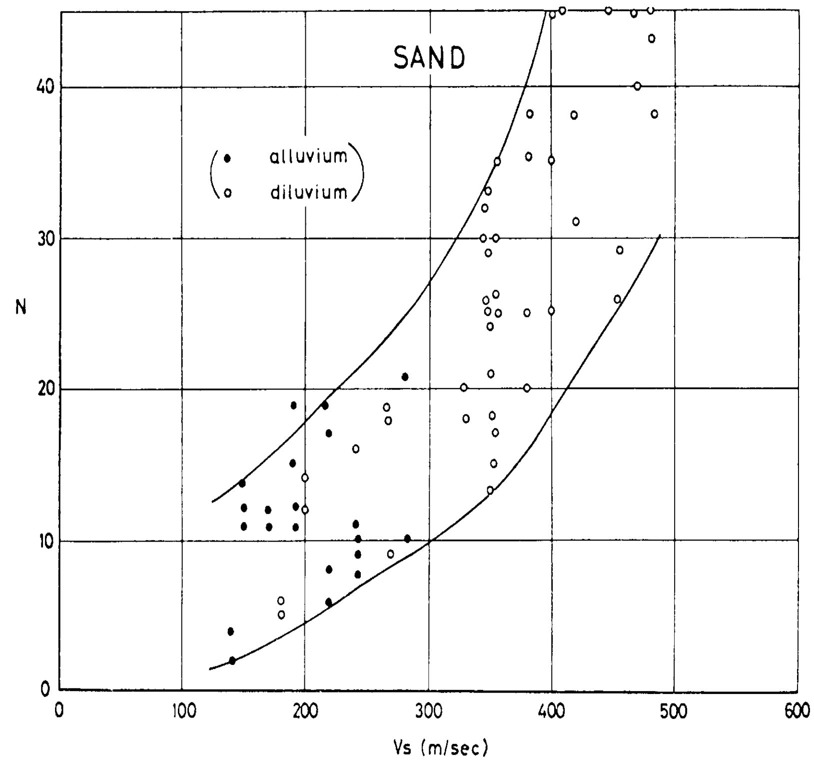
Relation between N-wave and S-wave velocity



Relation between N-value and S-wave velocity

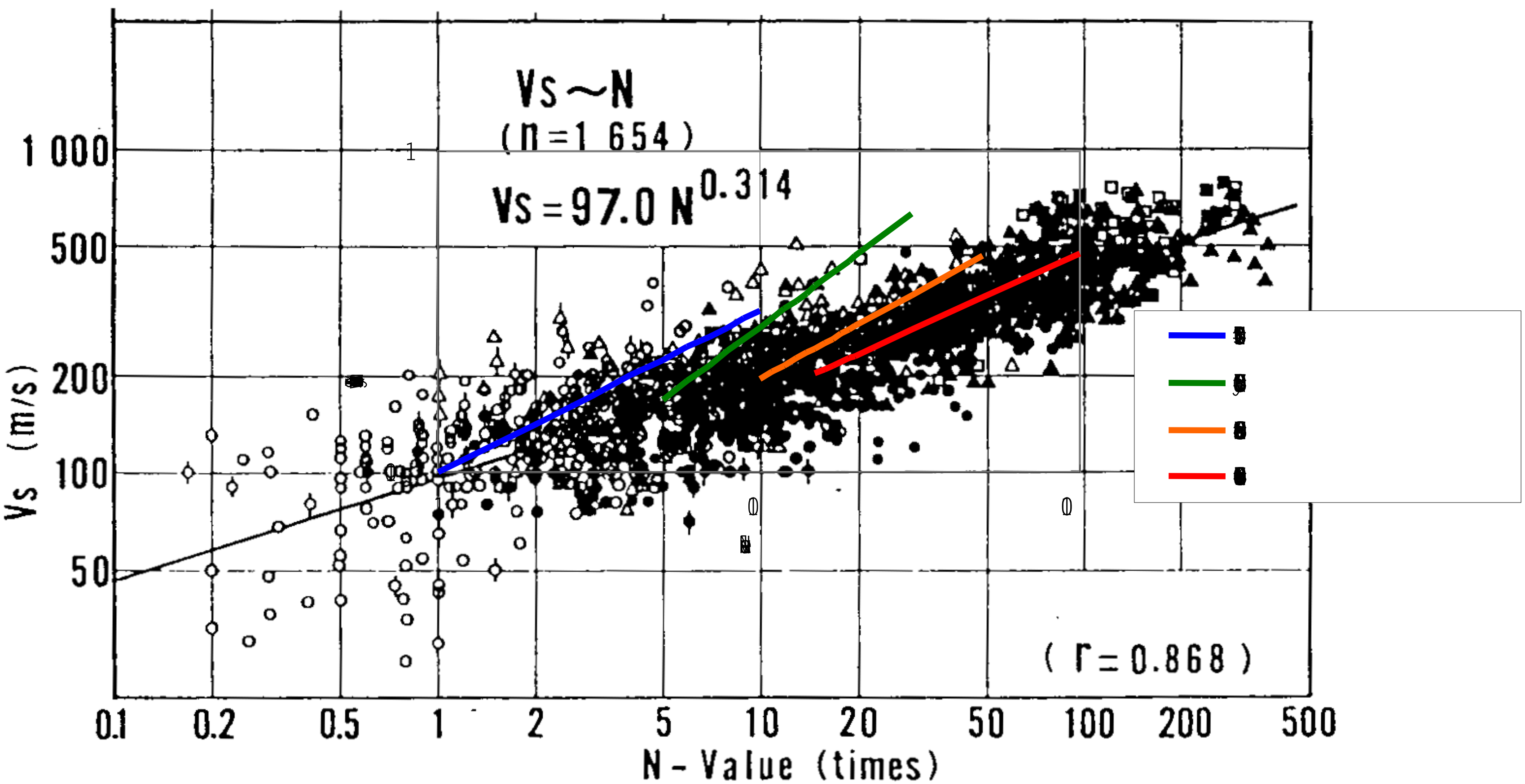


Silt and clay



Sand

Relation between N-value and S-wave velocity



Summary□Characteristics of the Surface-wave Methods

- Phase velocity of surface-waves is sensitive to the S-wave velocity
- Both active and passive surface waves can ve used
- You can obtain S-wave velocity of the ground easily with surface-wave methods
- You don’t need dynamite or special vibrators