Utilization of high-frequency Rayleigh waves in near-surface geophysics

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Rayleigh waves, surface waves that travel along a "free" surface such as the earth-air or the earth-water interface, are usually characterized by relatively low velocity, low frequency, and high amplitude. Rayleigh waves are the result of interfering P and SV waves. Particle motion of the fundamental mode of Rayleigh waves in a homogeneous medium moving from left to right is elliptical in a counterclockwise (retrograde) direction along the free surface. As depth increases, the particle motion becomes progaded and is still elliptical when reaching sufficient depth. The motion is constrained to a vertical plane consistent with the direction of wave propagation.

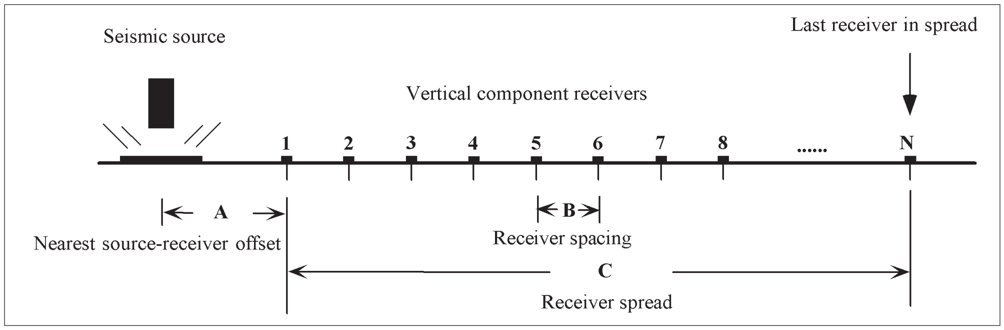

Figure 1. *Three acquisition parameters. (a) The nearest source-receiver offset is approximately equal to the maximum investigation depth. (b) Receiver spacing = the thinnest layer of the layer model. (c) Receiver spread distance between the first receiver and the last receiver = two times the maximum investigation depth.*

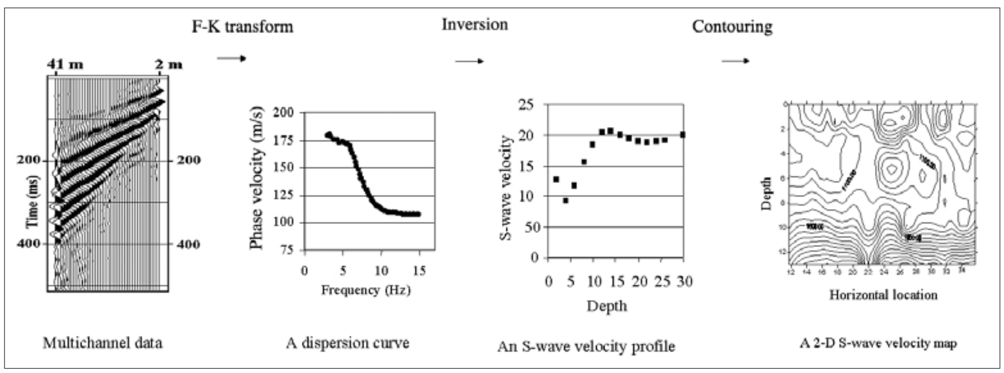

Figure 2. *A diagram of the MASW method. Multichannel raw-field data, which contain enhanced Rayleigh-wave signals, are first acquired. Rayleigh-wave phase velocity is then extracted from the field data in the f-v domain. The phase velocity, finally, is inverted for a shear-wave velocity profile (V_s versus depth). If a number of multichannel records in a standard CDP roll-along acquisition format are collected, a 2D S-wave velocity section can be generated.*

In the case of a solid homogenous half-space, the Rayleigh wave is not dispersive and travels at a velocity of approximately 0.9194 v when Poisson's ratio is equal to 0.25 and where v is the S-wave velocity in the half space. However, in the case of one layer over a solid homo-genous half-space, Rayleigh waves become dispersive when their wavelengths are in the range of 1-30 times the layer thickness.

Longer wavelengths penetrate greater depths for a given mode, generally exhibit greater phase velocities, and are more sensitive to the elastic properties of the deeper layers. Conversely, shorter wavelengths are sensitive to the physical properties of near-surface layers. Therefore, a particular mode of surface wave will possess a unique phase velocity for each unique wavelength, leading to the dispersion of surface waves.

Shear-wave velocities can be derived from inverting the dispersive phase velocity of the surface (Rayleigh and/or Love) wave. Near-surface S-wave velocity can be determined by inverting high-frequency Rayleigh waves using a process that is called multichannel analysis of surface waves (MASW). This process includes acquisition of high-frequency (>2 Hz) broad-band Rayleigh waves, efficient and accurate algorithms designed to extract Rayleigh-wave dispersion curves from Rayleigh waves, and stable and efficient inversion algorithms to obtain near-surface S-wave velocity profiles.

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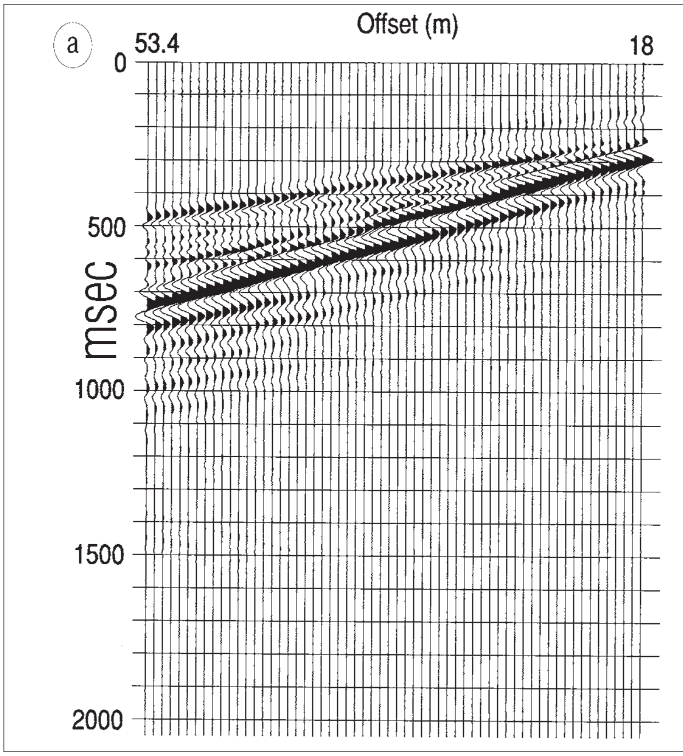
Near-surface S-wave velocities. MASW estimates S-wave velocity from multichannel vertical component data and consists of three parts: data acquisition, dispersion-curve picking, and inversion. A 2D S-wave velocity section can be generated when surface wave data are acquired in a standard CMP roll-along acquisition format.

1) Surface-wave data acquisition. Optimal recording of Rayleigh waves requires field configurations and acquisition parameters favorable to recording planar Rayleigh waves. Depending on investigation depth, Rayleigh waves of certain lengths need a specific amount of time to be developed into planar waves. Plane-wave propagation of surface waves does not occur in most cases until the near-offset (distance between the source and the first receiver) is greater than half the maximum desired wavelength. The maximum penetration depth in a homogeneous medium is about one wavelength. The currently accepted rule of thumb for the maximum penetration depth is approximately half the longest wavelength. The nearest receiver to source offset distance should be almost the same as the principle investigation depth. High-frequency surface waves attenuate quite rapidly with distance away from the source. To record high



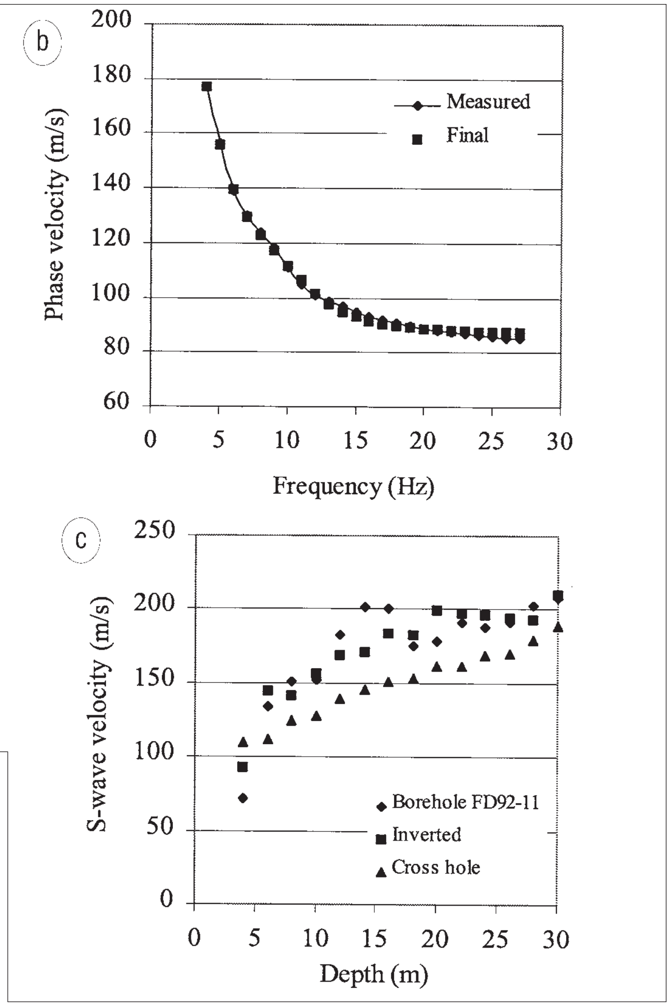


Figure 3. *Well FD92-11, one of eight examples from Fraser River Delta, Vancouver, Canada. Raw data (a) were acquired using a 60-channel seismograph with 4.5-Hz vertical-component geophones that were deployed at 0.6-m intervals with the nearest offset of 18 m. The source was a weight drop. (b) Derived dispersion curves. (c) Inverted S-wave velocity models compared with direct-borehole (diamonds) and crosshole (triangles) measurements.*

frequencies at the further offsets, the far-offset distance (the distance between the source and the furthest receiver) is optimally twice the investigation depth. A dispersion image in the frequency-velocity (*f-v*) domain can be affected by the geophone spread. Normally, the longer the geophone spread, the higher the resolution of the dispersion image. To avoid spatial aliasing, the receiver spacing should be less than half the shortest measured wavelength (Figure 1). Theoretical analysis by Zhang et al. supports our suggestions for field parameters.

2) Dispersion curves. Dispersion curves can be obtained using the transformation discussed by Park et al. (1998). The 2D transformation basically maps a shot gather into the *f-v* domain. A locus along peaks of dispersion energy over different frequency values in the *f-v* domain permits the construction of dispersion curve images. The resolution of the peaks is key to an accurate dispersion curve, which is critical to the next step: inverting phase velocities to obtain the S-wave velocity profile.

3) Inversion of dispersion curves. The Rayleigh-wave phase velocity of a layered earth model is a function of frequency and four earth properties: P-wave velocity, S-wave velocity, density, and thickness of layers. Analysis of the Jacobian matrix provides a measure of dispersion-curve sensitivity to these earth properties. Shear-wave velocity is the dominant influence on a dispersion curve in the high-frequency range (>2 Hz), therefore only S-wave velocities are considered unknowns in the inversion. An iterative solution to the weighted equation (Xia et al., 1999) proved very effective in the high-frequency range when using the Levenberg-Marquardt (L-M) method. Convergence of the solution is guaranteed and stable through selection of an initial model and the damping factor of the L-M method.

4) 2D S-wave velocity sections. If surface wave data are collected in a CMP roll-along acquisition fashion, a 2D S-wave velocity section can be generated with gridding software by placing each S-wave profile (V_s versus depth) in the middle of the geophone spread with which it was calculated. This is a relatively low horizontal resolution section. In the last section of this paper we will introduce a method to improve the horizontal resolution of this section.

Figure 2 shows the processing flow from a shot gather to an S-wave velocity profile and then to a 2D S-wave velocity section for multishots acquired in the CMP roll-along fashion. Shear-wave velocity profiles derived from MASW compared favorably to direct borehole measurements at sites in Kansas, British Columbia (Figure 3), and Wyoming (Figure 4). On average, the difference between MASW-calculated V_s and borehole-measured V_s is less than 15%. The MASW method not only provides accurate near-surface S-wave velocities but in some geologic settings it is the only surface seismic way to obtain S-wave velocity information. Examples are a dipping layer where converted P-wave velocity could occur in a shear-wave refraction survey or in the case of velocity inversions (a higher velocity layer on the top of a lower velocity layer), which precludes refractions event from returning from the interface.

Utilization of higher modes. A series of different-frequency Rayleigh waves can have the same apparent phase velocity. These different-frequency Rayleigh waves with a given phase velocity are known as modes and are characterized by their different number of horizontal nodal planes (planes

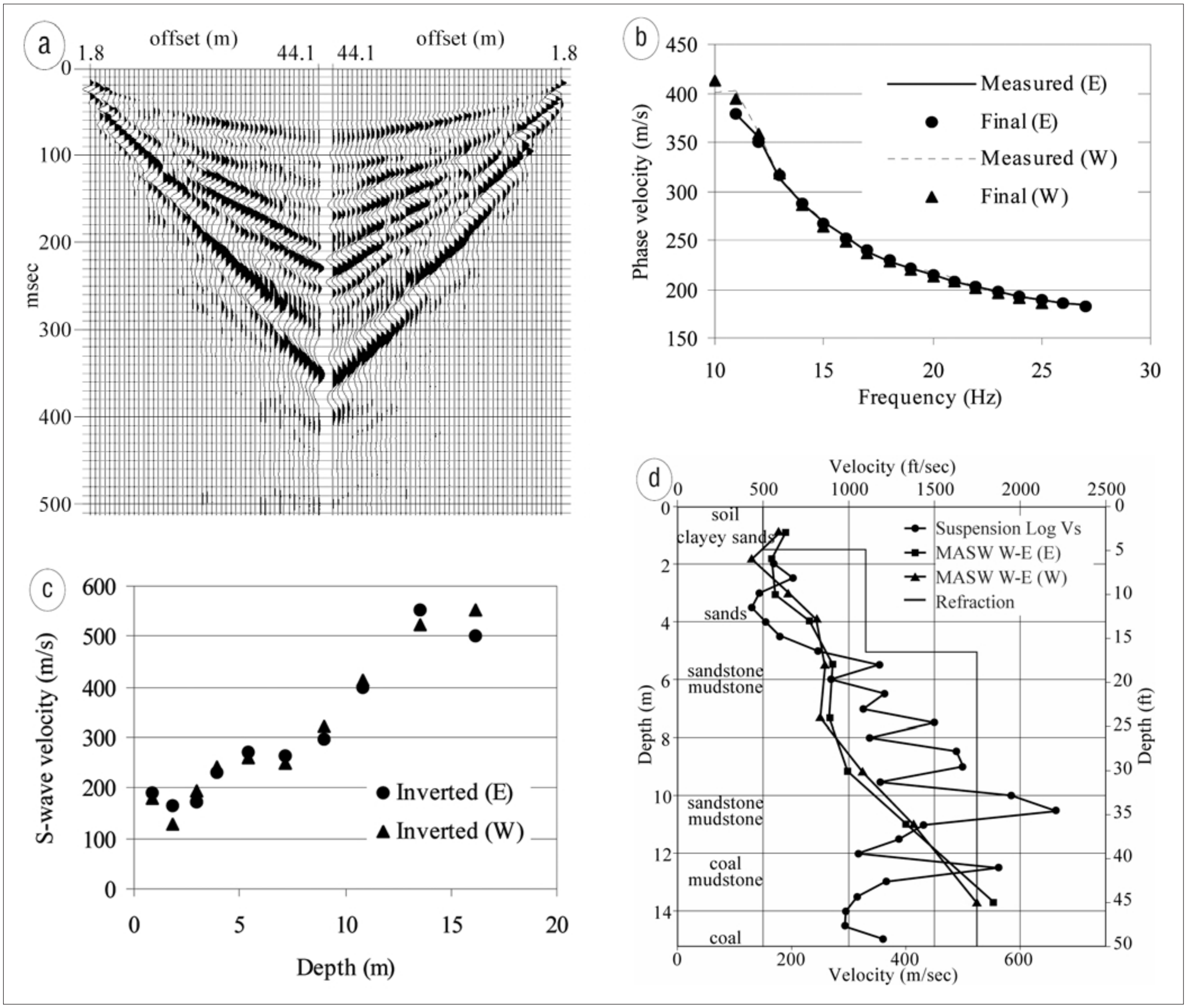


Figure 4. *(a) Raw data from a mining site in Wyoming. Data were acquired from both ends of the geophone spread. After a shear-wave refraction survey produced converted P-wave velocities, surface-wave data were acquired using a 48-channel seismograph with 8-Hz vertical-component geophones that were deployed at 0.9-m intervals with the nearest offset of 1.8 m. The source was a 6.3-kg hammer vertically impacting a metal plate. Processed results were shown by dispersion curves (b) and inverted S-wave velocity models (c). Labels E and W denote whether seismic source is at the east or the west end of the line. The inverted S-wave velocity models (c) were compared with S-wave velocities from a suspension log (d) and results from the refraction survey.*

of no particle displacement within the layer). In other words, because these waves can travel at different velocities for a given frequency, more than one phase velocity can be associated with a given frequency of Rayleigh wave. The lowest velocity for any given frequency is called the fundamental mode velocity (or the first mode). The next velocity higher than the fundamental mode phase velocity is called the second mode velocity, and so on. All phase velocities that are higher than the fundamental mode velocities are called higher modes.

The inversion process is more stable when higher mode data are included. Experimental analysis indicates that higher mode energy tends to become more dominant as the source-to-receiver distance increases. In some cases, analysis of higher mode data is necessary because shorter wavelength components of fundamental mode Rayleigh waves are obscured by these higher Rayleigh wave modes. Higher mode data have a deeper investigation depth than fundamental mode data. Modeling results and real-world examples show that higher mode data stabilize the inversion procedure and increase the resolution of inverted S-wave velocities.

High-frequency surface-wave data were acquired in San Jose, California, to determine shear-wave velocities of near-surface materials up to 10 m deep. Higher mode data were used to obtain a stable S-wave velocity profile. Raw data (Figure 5a) were acquired by using a 60-channel Geometrics StrataView seismograph. Thirty 4.5-Hz vertical-component geophones were used with a 1-m geophone interval and a nearest source-to-receiver offset of 4 m. Note on the dispersion-curve (Figure 5b) where high modes are obvious. The second mode is from 20 to 50 Hz and the third mode starts at 35 Hz. Three data sets were generated and inverted for comparison. The first set was fundamental-mode surface-wave data only (Figure 5c), automatically extracted from the dispersion data (Figure 5b). The second data set was fundamental-mode data with noise deliberately introduced in the frequency range from 13 to 19 Hz (Figure 5d). Noises were experimentally generated to simulate a case where the fundamental-mode data are contaminated with higher modes and/or body waves. Based on our experience, the shape of the second data set as shown here is commonly seen in real dispersion curves. The standard deviation between these two data sets is only 16 m/s.

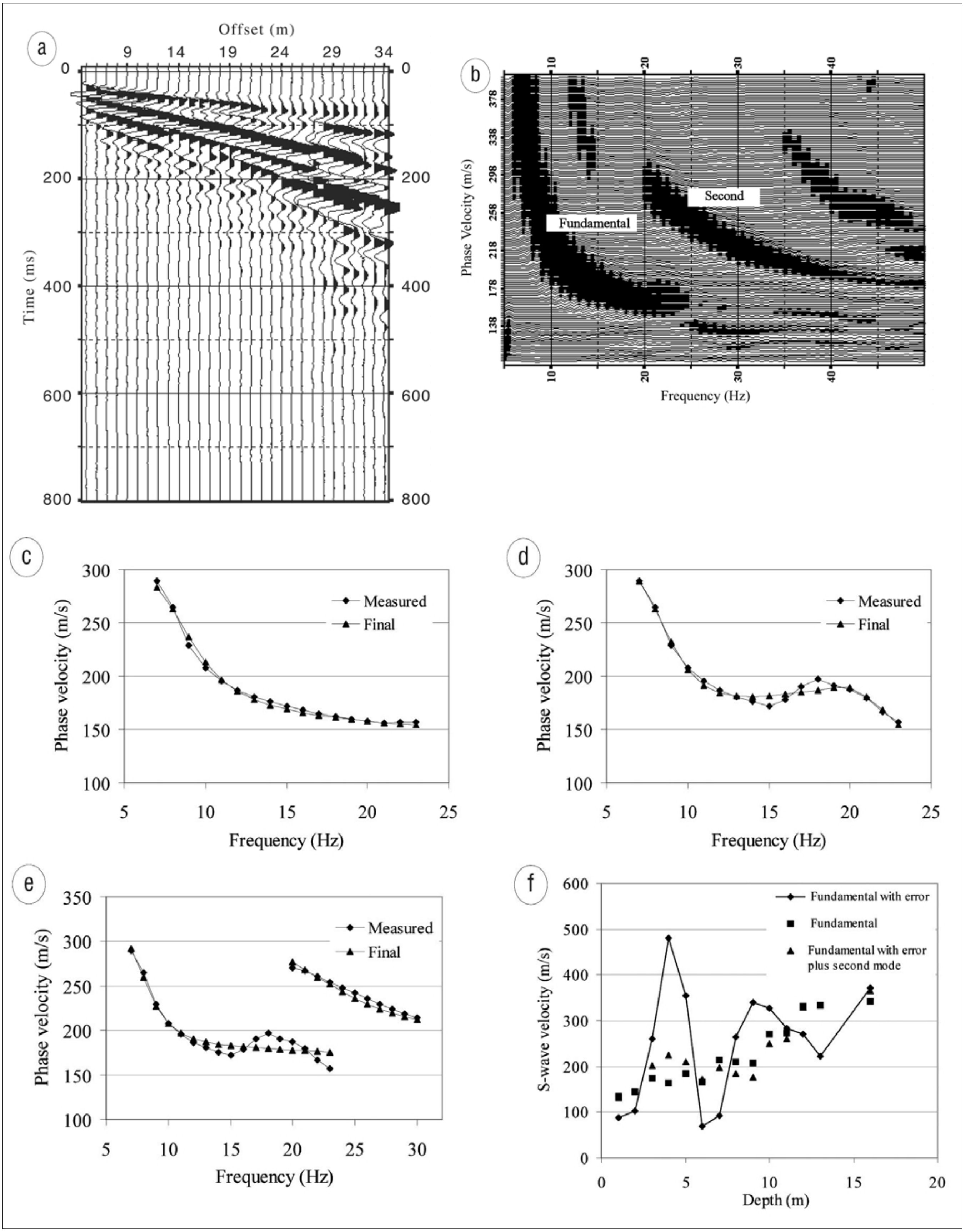


Figure 5. *An example from San Jose, California. (a) Raw data. (b) Dispersion curve image. (c) The fundamental mode phase velocities. Data labeled "measured" are extracted from (b) and those labeled "final" are calculated based on the shear-wave velocity model (solid squares in Figure 5f). (d) The fundamental-mode phase velocities with errors. Data labeled "measured" are extracted from (b) with noise deliberately introduced and the "final" data are calculated based on the shear-wave velocity model (diamonds with a solid line in Figure 5f). (e) The erroneous fundamental mode and the second mode phase velocities. "Measured" are the same as (d) and higher mode data from 20 Hz to 30 Hz are extracted from (b). "Final" are calculated based on the S-wave velocity model (solid triangles in Figure 5f). (f) Inverted S-wave velocity profiles.*

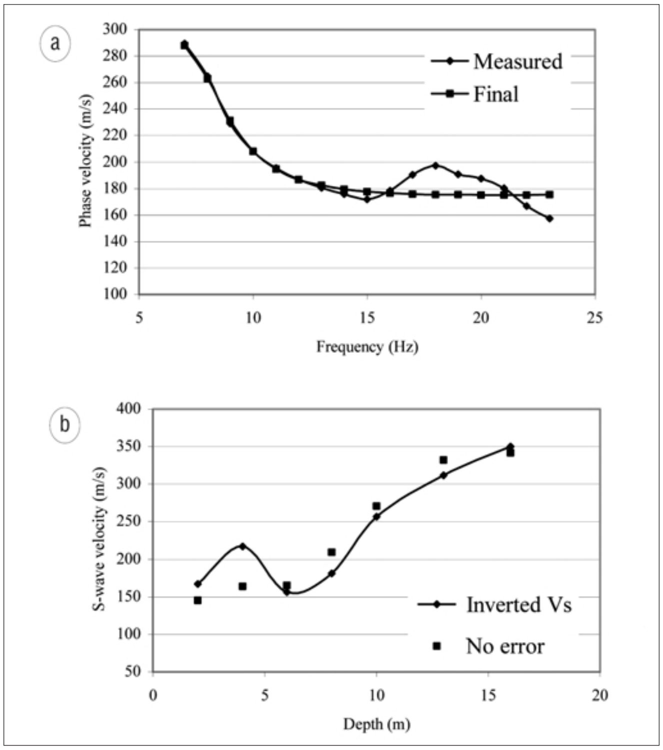


Figure 6. *The example shown in Figure 5 with reduced resolution. Phase velocities labeled "measured" with diamonds (a) are the same as data labeled with diamonds in Figure 5d. Data labeled "final" with squares are calculated based on a stable and smoothed shear-wave velocity model (diamonds with a solid line in Figure 6b). Solid squares in (b) are the same as the solid squares in Figure 5f.*

The third data set included the second set (noisy data) and the second mode surface wave data (Figure 5e). A 14-layer model with each layer 1 m thick was chosen to test these three data sets.

All root-mean-square (rms) errors between the measured dispersion curves and calculated dispersion curves (Figures 5c-e) from each of these S-wave velocity models (Figure 5f) are less than 5 m/s. Because the fundamental-mode data (Figure 5c) were accurately extracted from Figure 5b, the inverted S-wave velocities (solid squares in Figure 5f) were geologically reasonable. They smoothly increase from shallower layers to deeper layers. However, smoothness disappears when the second data set (Figure 5d) was inverted. The S-wave velocity model (diamonds with a solid line) changes irrationally in the depth range 3-7 m. This instability is caused by forcing the response of the inverted model to fit the error.

In the real world, it is common to provide an error range that will force an inverted model into an unreasonable space. We have experienced this situation a number of times when processing surface wave data. Better results are obtained when higher mode surface wave data (Figure 5e) are inverted simultaneously with the fundamental-mode data. Because of the higher rms error in the calculated second mode data, the S-wave velocity model with abrupt variation (diamonds with a solid line) was rejected. Inverted S-wave velocities (solid triangles) that included second modes in the inversion were similar to results obtained from data set one (solid squares). The inversion is more stable when including higher mode data in the inversion of surface wave data. This stability indeed improves the resolution of inversion results.

So what should we do if no higher modes are available? We have to make a choice between error and resolution of the inverted model. A trade-off between resolution and error of a model to obtain stable results is a wise strategy. We can reduce errors in the inverted S-wave velocity model by reducing the resolution of the model (increasing thickness of layers). In the San Jose example, we inverted data set 2 (Figure 5d) again with a seven-layer model, each layer being 2 m thick. This model possesses only half the resolution of the previous model (1 m thick in Figure 5f). Data set 2 (diamonds with a solid line in Figure 6a) underwent the same inversion procedure used for the San Jose example (Figure 5). Clearly, the inverted S-wave velocity model with the reduced resolution (diamonds with a solid line Figure 6b) was smoother and geologically more acceptable than the inverted mode depicted by diamonds with a solid line (Figure 5f).

Near-surface Q. The quality factor (Q) as a function of depth is of fundamental interest in groundwater, engineering, and environmental studies, as well as in oil exploration and earthquake seismology. A desire to understand the attenuative properties of the earth is based on the observations that seismic-wave amplitudes are reduced as waves propagate through an elastic medium. Modeling results suggest that it is feasible to solve for P-wave quality factor Q_P and S-wave quality factor Q_S in a layered-earth model by inverting Rayleigh-wave attenuation coefficients when V_s/V_p reaches 0.45. Only Q_S can be estimated from Rayleigh-wave attenuation coefficients when V_s/V_p is less than 0.45. Sensitivity analysis showed that errors in inverted quality factors can reach 1-1.5 times the error in associated attenuation coefficients. Compared to the inversion system for Rayleigh waves (10% error in surface-wave phase velocity will result in 6% error in S-wave velocity), the inversion system for Q has less stability. Hence, accurate calculation of Rayleigh-wave attenuation coefficients is critical. On the other hand, the inversion system for Q is more stable than AVO routinely practiced in the oil industry. It is known that in AVO analysis that a 10% error in incident angles could result in a 40% error in reflection coefficients. By introducing a damping factor, Q_P and/or Q_S can be solved from attenuation coefficients of Rayleigh waves.

Surface-wave data (Figure 7a) were acquired by a 60-channel seismograph with 4.5-Hz vertical-component geophones that were deployed at 1.2-m intervals with the nearest offset of 4.8 m. S-wave velocities (Figure 7b) of a 10-layer model were calculated by MASW with known P-wave velocities.

Attenuation coefficients of Rayleigh waves (Figure 7c) were calculated by the definition of amplitude attenuation. "Measured" data were calculated from raw data and those labeled "final" were calculated based on the inverted quality factor model. Q factors in up to 20 m (Figure 7d) were found based on the Rayleigh-wave attenuation coefficients (Figure 7c). Q_S was calculated to be in the range of 7-25. Q_P was assumed twice as large as Q_S. Modeled Rayleigh-wave attenuation coefficients ("final" in Figure 7c) are a good match with the measured coefficients.

Resolution of S-wave velocity model. Understanding the model resolution in the inversion of Rayleigh-wave phase velocities is critical in applying MASW to near-surface geological/geophysical problems. The model resolution matrix indicates that a model can be perfectly resolved in a least-squares sense if error-free data are inverted. Errors can be introduced during data acquisition and/or by artifacts created during data processing. Errors in data are equivalent to a smear matrix in the model space that reduces the model resolution. The degree of smear in a model depends on data accuracy. Because of the smear matrix, an S-wave velocity model obtained by inversion of Rayleigh-wave phase velocities in the real world cannot be perfectly resolved. However, the resolution power of Rayleigh-wave data can be ascertained after determining the data accuracy. Model resolution can be increased by increasing the accuracy of measured data.

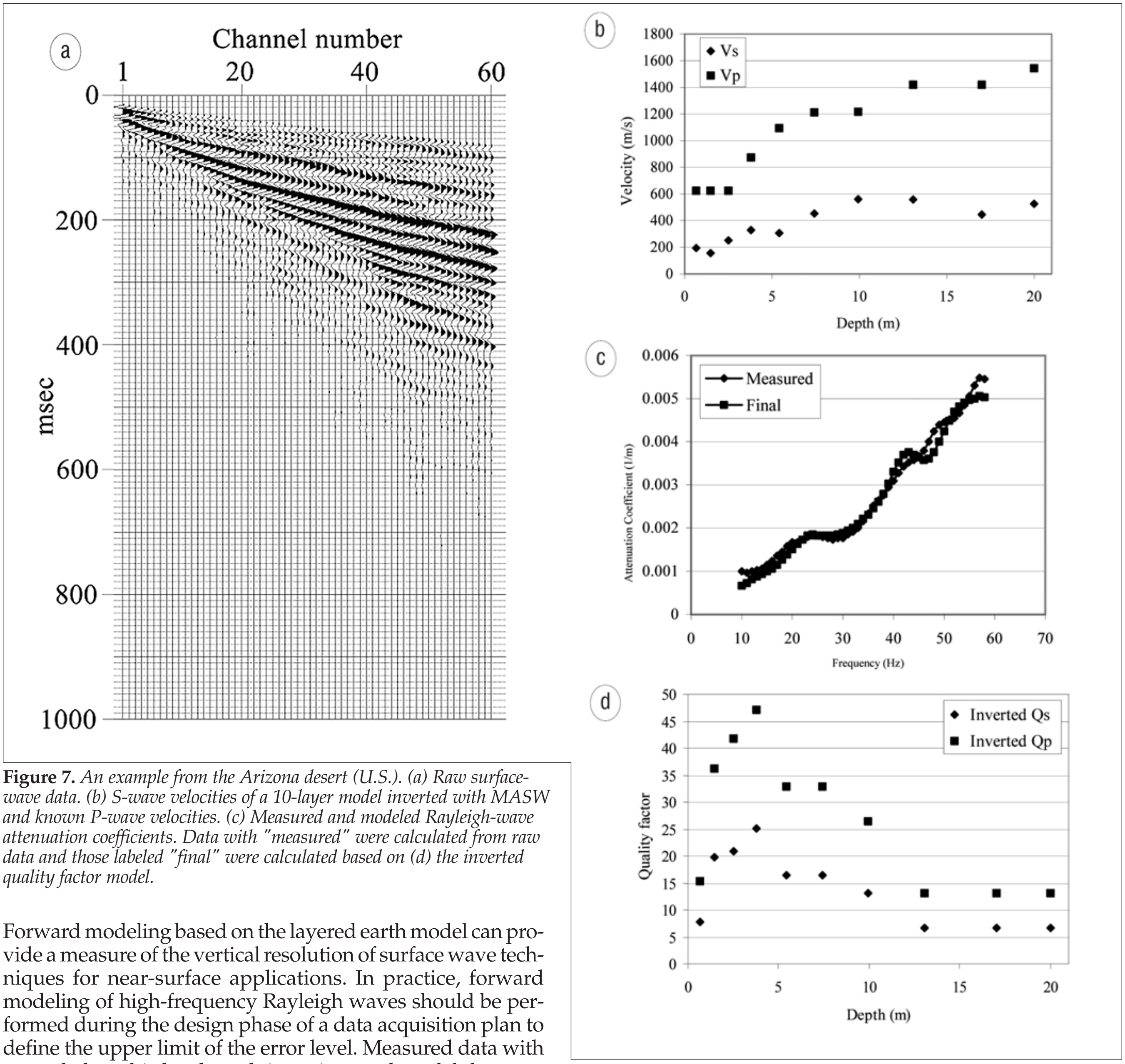


Figure 7. *An example from the Arizona desert (U.S.). (a) Raw surface-wave data. (b) S-wave velocities of a 10-layer model inverted with MASW and known P-wave velocities. (c) Measured and modeled Rayleigh-wave attenuation coefficients. Data with "measured" were calculated from raw data and those labeled "final" were calculated based on (d) the inverted quality factor model.*

Forward modeling based on the layered earth model can provide a measure of the vertical resolution of surface wave techniques for near-surface applications. In practice, forward modeling of high-frequency Rayleigh waves should be performed during the design phase of a data acquisition plan to define the upper limit of the error level. Measured data with errors below this level result in an inverted model that possesses a vertical resolution sufficient for geologic interpretation. Feeding poor data into inverse algorithms can only produce unrealistic models.

When surface wave data are acquired in the CMP roll-along format, a 2D S-wave velocity section can be generated by placing on S-wave velocity profile in the middle of each geophone spread. This section provides a blurred image of the true S-wave velocity at that point due to the width of geophone spread. However, with the redundancy in coverage using the CMP roll-along acquisition method, an unblurred picture of S-wave velocity can be obtained using the generalized inversion. Assume **c** is a vector of S-wave velocities at a given depth (**c** is blurred S-wave velocities) and **s** is a vector of unblurred S-wave velocities at the depth **c** = **Gs**, where **G** is the data kernel or a weighting matrix that could be determined by a user or accuracies of each elements of blurred S-wave velocities **c**. The system **c** = **Gs** is undetermined. The minimum length solution can be found by a generalized inversion as **s** = **GT [GGT]**$^{-1}$c. Stability in the generalized inverse matrix can be achieved by introducing a damping factor. To smooth the inverse, we selected, with no firm theoretical basis, the singular value of matrix **G** where the second derivative reaches the highest value as a damping factor. There are probably other ways to determine the damping factor. The horizontal resolution can be improved by this extra processing step.

Other applications and development. In petroleum exploration, a near-surface layer acts as a filter that smears images of deep reflection events. Accurate near-surface velocity information is critical to eliminate the smearing effect. However, determining near-surface velocities is a troublesome task, especially for S-wave reflection survey. One promising alternative is to use MASW. It can accurately determine S-wave velocities for a two-layer model, which is a basis for estimating static corrections in a S-wave survey.

In environmental studies, 2D shear-wave velocity fields calculated from inversion of Rayleigh-wave phase velocities were successfully used to define bedrock interfaces and near-surface geologic structures from 2 to 50 m and determine a collapse feature in an extremely noisy environment. Fast and efficient methods for geophone deployment—autojuggie and landstreamers, have been utilized with the

MASW method. In construction engineering, one direct application of the surface-wave technique is to find the N-value, an index value of formation hardness used in soil mechanics and foundation engineering.

Conclusions and future studies. Shear-wave velocity information is critical in geophysical and geological applications. Accurate S-wave velocities (±15%) can be obtained by inverting Rayleigh-wave phase velocities. The inversion system is numerically stable. An inversion system with higher mode data provides even greater stability and improved resolution in the inverted model. It also is feasible to determine near-surface quality factors Q_P and/or Q_S from attenuation coefficients of Rayleigh waves. Future studies of high-frequency surface waves in near-surface geophysics include improvement of the dispersion image in the *f-v* domain, determination of S-wave velocities from group velocity of Rayleigh waves, physical and numerical modeling of surface-wave response in various geologic settings, inversion of Love waves, a joint inversion of Rayleigh and Love waves, and 3D surface-wave techniques.

Suggested reading. "Geophysical Data Analysis-Discrete Inversion Theory" by Menke (Academic Press, 1984). "Imaging dispersion curves of surface waves on multichannel record" by Park et al. (*SEG 1999 Expanded Abstracts*). "Toward the autojuggie: Planting 72 geophones in 2s" by Steeples et al. (*Geophysical Research Letters*, 1999). "Estimation of near-surface shear-wave velocity by inversion of Rayleigh wave" by Xia et al. (*GEOPHYSICS*, 1999). "Determining Q of near-surface materials from Rayleigh waves" by Xia et al. (*Journal of Applied Geophysics*, 2002). "Increasing horizontal resolution of geophysical models by generalized inversion" by Xia et al. (*SEG 2004 Expanded Abstracts*). "The selection of field acquisition parameters for dispersion images from multichannel surface wave data" by Zhang et al. (*Pure and Applied Geophysics,* in press). TLE

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