

QUANTITATIVE MAGNETIC ANALYSIS
OF LANDFILLS)

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Summary

Magnetic surveys can furnish an estimate of the number of buried steel drums at hazardous waste landfills. This procedure could be valuable when records and recollections are inadequate. While a magnetic survey will only allow an approximation to the quantity of drums, it will probably be more accurate than any other geophysical technique. Its advantage is that no excavation is needed; its major disadvantage is that other iron debris can degrade the accuracy of the analysis.

The points discussed here are the magnetic patterns of landfills and individual drums, and the procedures for doing a magnetic survey and interpreting the results.

Quantitative Magnetic Analysis of Landfills

Introduction

The best geophysical technique for locating and mapping the distribution of iron is a magnetic survey. Since hazardous wastes are often buried in steel drums, a magnetic survey can provide indirect clues to their location. However, in addition to finding drums, it is valuable to be able to determine the quantity of buried chemicals at a landfill. With a careful magnetic survey and analysis, an approximation to the quantity of buried iron is possible. While the estimation of the number of drums is indirect and has several difficulties, it can still be an aid in planning remedial action at a landfill.

Magnetic Maps of Landfills

Iron is a good "conductor" of magnetism. The earth's natural magnetic field will tend to concentrate in iron objects. Figure 1 is an example. To make the important points clear, this is the calculated magnetic map of an orderly landfill. It consists of about 8000 steel drums over a rectangular area of 80 by 120 feet; the layers of drums begin at a shallow depth and extend to a depth of 10 feet.

The oval lines in the figure indicate the changes in the earth's magnetic field when measurements are made over this mass of iron. The earth's field increases by 8% over the landfill; the field decreases outside the landfill, in particular on the northern side.

This is a typical and characteristic magnetic pattern. The unit of the magnetic field is called the nanotesla, honoring the American engineer Tesla, and is abbreviated nT. An earlier name for an equivalent unit is the gamma. Figure 1 indicates that the earth's field increases by over **4-500** nT above its normal value of **55 000** nT. Since magnetic measurements can easily be made to a precision of 1 nT, this quantity of iron is easy to detect.

Since the density of the earth's magnetic flux is increased in the area of the landfill, it must decrease in other areas. The principal area of reduction is on the northern side of the

landfill and is marked by a stippled pattern in Figure 1. This magnetic low is an important part of the magnetic map, for its amplitude aids the determination of the quantity of iron in the landfill.

The magnetic high, however, is the most important feature. Note that the magnetic contours give only a blurred image of the shape and location of the iron mass.

The magnetic maps of actual landfills are more complex than the example of Figure 1, but also have general similarities. The magnetic map of part of a municipal landfill is given in Figure 2. The peak of the magnetic field, or anomaly, is about 1500 nT above the earth's normal field in this area; this is still a very intense anomaly indicating a large quantity of buried iron. The iron is concentrated in the areas having a high magnetic field. Since this is a municipal landfill with the likelihood of considerable iron-containing trash, the magnetic survey locates pockets of iron without identifying them as iron drums or not. While imperfect, this delineation can greatly reduce the amount of excavation required.

As was the case in Figure 1, there are magnetic lows (stippled) on the north side of the magnetic highs. This high-low pattern is usual and indicates a distinct concentration of iron. This high-low pattern is not as clear in the magnetic map of Figure 3. This is part of another municipal landfill. This landfill is in the form of a plateau; the northern edge of the survey area reached the edge of this plateau and the iron which is distributed through the landfill causes the large area of low magnetic field on the north side of the survey area.

Here again, the islands of high magnetic field approximately locate concentrations of iron. While buried drums were found at one of these areas, normal iron refuse caused some of the other magnetic anomalies.

The surveys illustrated in Figures 2 and 3 were primarily for the purpose of locating buried iron rather than trying to estimate the quantity of buried drums which might be in the landfills. However, these surveys do illustrate the magnetic patterns of rather complex distributions of iron. Many hazardous

waste landfills can have little household or other industrial iron trash within them, and the magnetic maps can therefore be more reliable indicators of buried steel drums.

The Magnetic Properties of Steel Drums

The magnetic pattern of a single steel drum was investigated. This drum was a typical variety, having a capacity of **55** gallons and a weight of 40 pounds. Its diameter was 1'11" and length was 2'11"; the circumference steel was 18 gauge and the ends were of 20 gauge steel.

A magnetically quiet area was found in a sand pit and a reference magnetic survey was made in a small area; the maximum change found in this area was 7 nT. The steel drum was placed in the middle of this area and a second magnetic survey was done. The differences between the readings of these two surveys indicates the magnetic field of the drum; these values are plotted in Figure 4. The peak anomaly is seen to be a magnetic high of 71 nT; the drum was at this point. The magnetic sensor was raised in the air to a height of 10 ft above the middle of the drum; the drum was clearly detected at this distance. In fact, this map suggests that the drum would be detectable even at a height of 20 ft.

The magnetic field of the drum depends of the drum's orientation, for two reasons. The first is that it is somewhat elongated in one direction; this orientation effect occurs for all objects except spheres. The second orientation effect is caused by permanent magnetism. In addition to "conducting" the earth's field, iron can also act like a permanent magnet. The first effect is called induced magnetization while the latter is known as remnant magnetization.

A test for remnant magnetization was made by turning the drum upside down and resurveying the area. As Figure **5** illustrates, the magnetic field pattern is very similar, although the anomaly amplitude has been reduced to **56** nT. While this test is by no means complete enough to be very accurate, it suggests that the ratio of induced to remnant magnetization could be 4:1. Most of the magnetic anomaly results from magnetic induction.

A third magnetic survey was done also, this time with the drum horizontal and aligned east-west. Figure 6 indicates that the anomaly is again similar, but reduced somewhat further in amplitude. In a landfill, the steel drums will be typically in a random orientation. This causes the remnant magnetization components to almost entirely cancel out; however, the components of induced magnetization add. The anomalies in the three surveys of Figures 4-6 can be averaged to give a better indication of the magnetic anomaly of a typical drum. This is Figure 7; because of the east-west symmetry, the halves of each survey map have also been averaged.

The magnetic field caused by objects can be calculated. The mathematical equation of this field is particularly simple for a compact object, ideally a sphere or spherical shell. While a sphere is not very similar to a cylinder such as a drum, their magnetic patterns can be very similar if measurements are not made too close to the cylinder or sphere. This simple source is known as a magnetic dipole. The magnetic anomaly of Figure 7 can be closely modelled with a magnetic dipole, as seen in Figure 8. Both the measurements and the model indicate a similar pattern and both have a weak, diffuse magnetic low on the north side.

The primary parameter of the dipole model is called the magnetic moment. This analysis indicates that the magnetic moment of the tested drum is about 34,000 nT*ft³. Other drums with different types of steel and, in particular, different thicknesses of steel, would have a different magnetic moment.

A related parameter is often more useful than the magnetic moment. This is called effective magnetic susceptibility and essentially quantifies how good an object is as a magnetic conductor. The magnetic susceptibility of iron varies over a wide range depending on its alloy; a value of about 12 might be considered typical (susceptibility is considered to be in c.g.s units here). From this fundamental parameter for iron itself, the effective susceptibility of a mass of drums in a landfill has been estimated in Figure 9. The effective susceptibility is based on an estimate of the volume percentage of iron in a dense landfill; while not necessarily very reliable, this extrapolation has some justification

in the relationship found between the volume fraction of the mineral magnetite in a rock and the rock's susceptibility (Grant and West 1965p. 366-69).

The effective susceptibility calculated in Figure 9 as 0.09 compares to that determined for the single tested drum; if a landfill were composed of those drums, the effective susceptibility would be about 0.05.

These estimates are only a beginning. A comparison of magnetic models and measurements of some subsequently-excavated landfills will allow more reliable estimates.

Field Procedures for a Magnetic Survey

While the direction of the earth's magnetic field is readily indicated with a compass, it takes a more elaborate instrument to accurately indicate the strength of the earth's field. This instrument is called a magnetometer. There are many varieties of this instrument; a common, practical version is called a proton magnetometer. The principles of these instruments and the general procedures for doing magnetic surveys can be found in several sources (Nettleton 1976p.327-59; Breiner 1973; Dobrin 1976p.485-518). Only some of the specific procedures for landfill surveys will be discussed here.

The most important point is that the magnetic sensor should be fairly high in the air. If it is wished to map the lateral distribution of iron in a landfill, a sensor height of 5-10 feet above the ground is suitable. However, for the most accurate estimates of the quantity of buried iron or drums, it is better that the sensor be 10-20 ft in the air. This is because the large mass of iron must be accurately mapped with minimal effect from small-area variations in the density of iron. A magnetic anomaly decreases rapidly with increasing height; for compact sources, this decrease is proportional to the inverse cube of the distance from the source. If the magnetic sensor is kept too low, magnetic variations due to shallow iron objects will obscure the large-scale pattern which is important. It is probably a good rule of thumb that the sensor should be at a height above the iron mass equal to twice the suspected thickness of the iron mass.

In principle, magnetic measurements made at a low altitude can be used to determine the magnetic field at a greater altitude by the method of upward continuation. In practice, the strong lateral gradients found at low altitudes will require very precise, accurately-positioned, and closely-spaced measurements for this mathematical extrapolation.

Magnetic measurements at a height of 10-20 ft are physically more difficult than measurements at a height of 5ft; trees and brush will add to the difficulty, for the sensor could be carried on the top of a wooden or aluminum pole. Measurements from a helicopter are possible, but it is then more difficult to control position. Position control would be easy with a helium-filled balloon (such as a kytoon) to carry the sensor to 20 ft or higher; this could not be done in wooded areas because of the necessary tethering cable. Fixed wing aircraft, while probably not suitable for detailed magnetic mapping, could be used in the search for lost or clandestine landfills; the magnetic anomaly of Figure 1 would be reduced to around 70 nT at an altitude of 100 ft and would be easily detectable.

The magnetic anomalies of many landfills will be sufficiently large that temporal corrections to the magnetic measurements will not be necessary. In some cases, it will be advisable to at least monitor the time change in the earth's field.

The analysis will be simplified if survey lines are measured along magnetic north-south directions. The along-line spacing of measurements can be about 10 ft. The spacing between parallel lines can be about 10-30 ft, depending on the spatial size of iron clusters in the landfill. The accuracy of positioning should be probably better than 10%. Pacing might be accurate enough if the end points of traverse lines can be well-fixed.

If a respirator is worn, its iron fittings will probably only affect the measurements if the sensor is below 10 ft in height. Other surface iron would be a greater difficulty.

Whenever possible, it would be better to do the magnetic survey before fencing a site. The magnetic survey could in fact help locate the desired area to fence. In order to analyze the magnetic data with as much accuracy as possible, it is advisable

to survey outside the area of the known drum concentration; in particular the magnetic low on the north side of the drum area should be mapped. This would be more difficult with interference from an iron fence.

Other surface structures can also cause strong magnetic anomalies. While steel-framed or roofed buildings are obvious, brick buildings can also cause strong anomalies. Iron trash or slag distributed around a landfill can cause unnecessary anomalies. While these unwanted sources of interference can be accounted for, they do complicate the analysis. Some electric trains can cause transients in the magnetic field.

The geology of the area of the landfill must be considered. While rare, veins of magnetite can give intense magnetic anomalies. Some igneous and metamorphic rock is magnetic also, but these rocks cause weak anomalies in comparison to those from iron.

Irregular terrain, with a relief greater than 10 ft, can make the survey and its analysis more difficult.

Some steels, in particular some types of stainless steel, are essentially nonmagnetic; since these metals are more expensive, it is unlikely that they would be used for disposal drums. Plastic drums are possible. Also, the toxic chemicals can be removed from drums. Most of these chemicals could not be directly detected with a magnetic survey. These limitations must be kept in mind.

For the direct detection of chemical concentrations, surveys with ground-penetrating radar, electromagnetic induction, or resistivity can be suitable; each technique has specific applications and limitations.

Radar surveys can help to define the lateral boundaries of landfills; Figure 10 is an example. It is not suitable for mapping the thickness of fill because the radar pulse is too strongly scattered within the fill. Resistivity soundings (Koefoed 1979 van Nostrand and Cook 1966p.86-110) might be able to determine the thickness of clean fill over the layer of drums. It may not be suitable for determining the thickness of the drum layer because of the high conductivity of that layer.

Resistivity and electromagnetic surveys can help to define

the lateral boundaries of landfills and can sometimes map the extent of leachate plumes.

Analysis of the Magnetic Maps

It would be desirable if the magnetic survey could directly indicate the thickness of the fill of steel drums. Figure 11 shows the magnetic profiles which could result from 40 ft wide landfills with two different thicknesses of steel drums. As expected, the thick layer causes a stronger anomaly, by a factor of three. Changes in the magnetic parameter, susceptibility, will also cause the anomaly amplitude to change. The shape of the two profiles can be compared by normalizing them, with the result in Figure 12. The shapes are quite similar; the profile difference on the south side of the fill would be essentially indistinguishable from a change in the width of the fill. The magnetic low on the north side of the thick fill is weaker, relative to the magnetic high, than the magnetic low for the thin fill. This small difference might be detectable, although with difficulty. A more distinctive difference is found in the area 40 ft north of the fill; this also might help estimate the thickness of the iron mass.

It is likely that anomaly shape alone will not be a reliable estimator of landfill thickness. It is probable that the anomaly amplitude must be used to indirectly indicate thickness by an estimate of the quantity of drums in the fill. This will require better knowledge of the magnetic properties of the steel drums in the fill.

The amplitude of the magnetic anomaly is determined by three factors. Their influence is shown in Figure 13. It is seen that, at the reference point, changes in any of the three parameters are about equally important in determining changes in the amplitude of the anomaly. Therefore, they must all be considered and controlled.

The primary work of magnetic interpretation is the approximation of the given magnetic data with models of iron distribution. There are two principal types of models. In one approach, each distinct magnetic anomaly is assumed to be caused by a buried polyhedron; this is most likely a simple shape such as a rectangular parallelepiped or a cylindrical prism. In the other approach, the anomaly is assumed to be caused by a cluster of buried spheres.

As models become more complex, their calculation becomes more time-consuming. Since computers will continue to get faster and cheaper, this factor will become less important.

If the magnetic anomaly extends for a distance to form a ridge, it is possible that its source can be modelled with a cylindrical or prismatic body extending in the direction of the ridge. These models are simple enough for a programmable calculator (Campbell 1981). A correction for the finite length of the prism may be necessary (Nettleton 1976 p. 381,2).

A three-dimensional polyhedron is much more difficult to describe and analyze. As a first step, there are sets of computed fields for basic shapes, such as the rectangular parallelepiped (Andreasen and Zietz 1969; Zietz and Andreasen 1967). The measured magnetic map might closely match one of these computed maps, and allow a rapid analysis.

If special shapes are necessary for the model, these can be analyzed, although a fast computer can be necessary since many iterations might be needed to get an adequate fit between the measured data and the calculated model (Talwani 1965 Kunaratnam 1981).

These polyhedron models are most suitable for simple magnetic anomalies. As the magnetic map gets more complex, it can be better to use the second class of models, the sphere cluster. The anomaly due to a sphere is simple (Parasnis 1979 p. 29-34; Smellie 1967). Figure 8 is an example. By using clusters of these sources and summations of their anomalies, complex magnetic patterns can be duplicated. These spheres will be in various locations and depths. Also, each one will have a distinctive magnetic parameter called its magnetic moment.

The result of the sphere cluster model will be the sum of the magnetic moments of the spheres. The result of the polyhedron model will be the volume of the source and its effective susceptibility. From either of these approaches, the number of drums or the weight of buried iron can be estimated by the procedure shown in Figure 14. The constants for converting the magnetic "volume" to the number of drums or the weight of iron are simply preliminary estimates and will have to be refined and tested for their variability from landfill to landfill.

There are fundamental ambiguities in the analysis of magnetic maps, for there is never one single model which will match the measurements. For example, a small volume with a high susceptibility can cause the same magnetic map as a larger volume with a lower susceptibility. Since the product of these two terms is all that is important, the uncertainty in the final result might be reasonable.

Conclusion

This is in fact just a beginning. The procedures suggested here may help the understanding of hazardous waste landfills and might guide the planning of their environmental safety. Geophysics can allow an estimate of the location, size, cover, and thickness of buried landfill materials. While the estimate has ambiguities and uncertainties, it can give some information without the dangers of excavation and coring.

Acknowledgment

The magnetic surveys which illustrate Figures 2 and 3 are included courtesy of the sponsors of these surveys. I appreciate being able to include these examples in this report.

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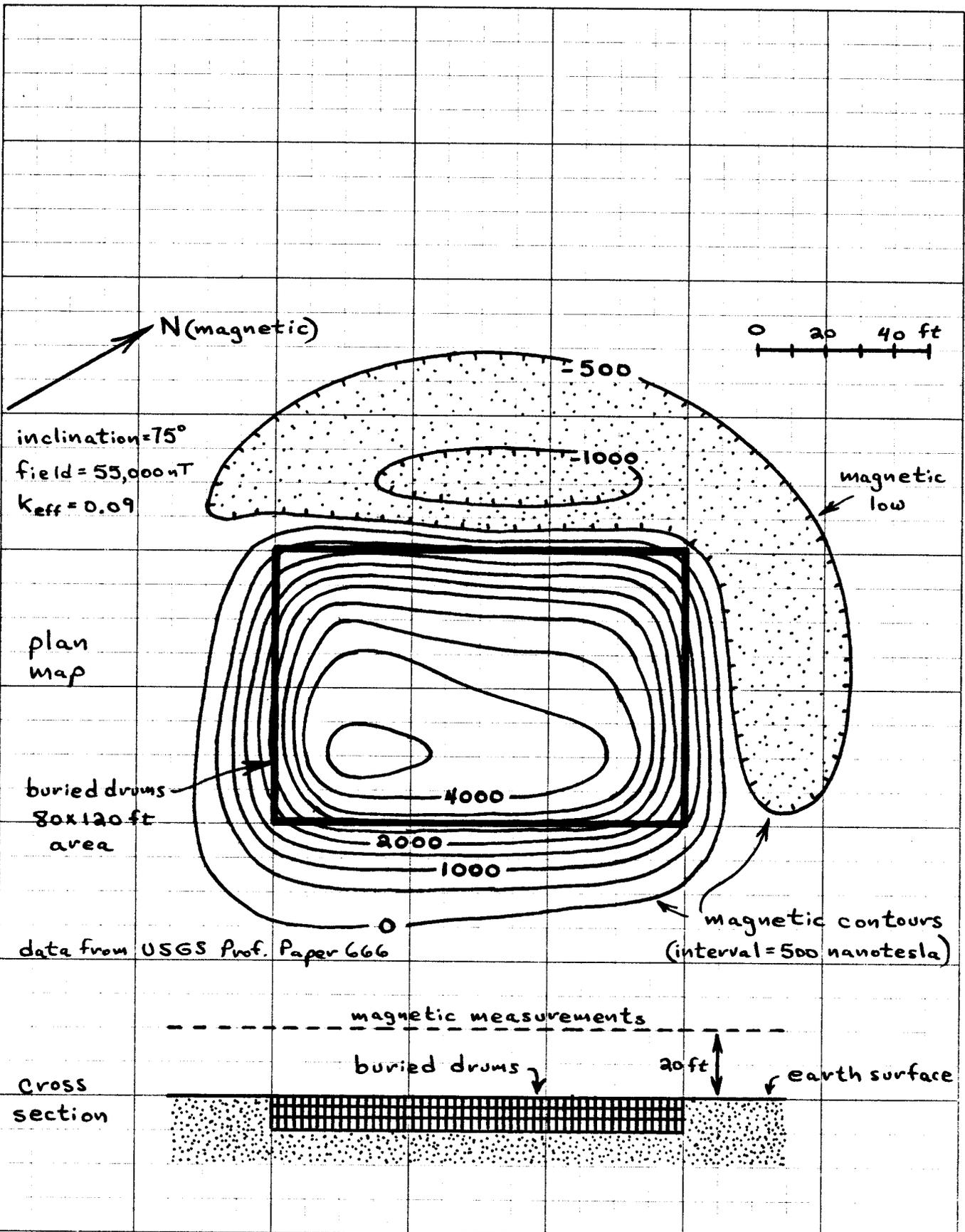
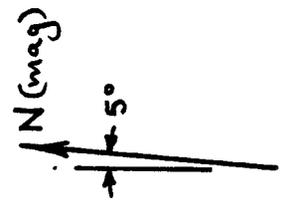
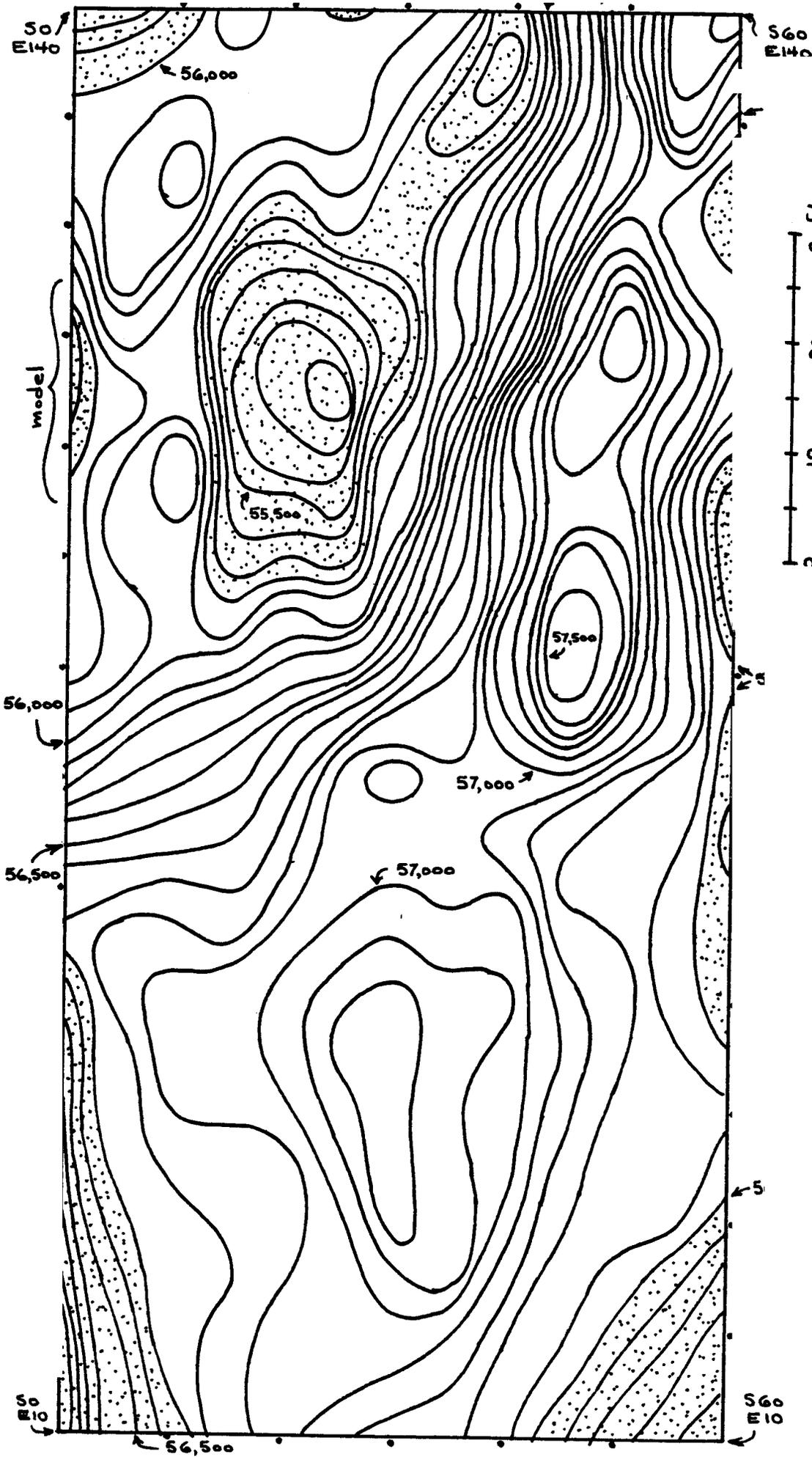


Figure 1

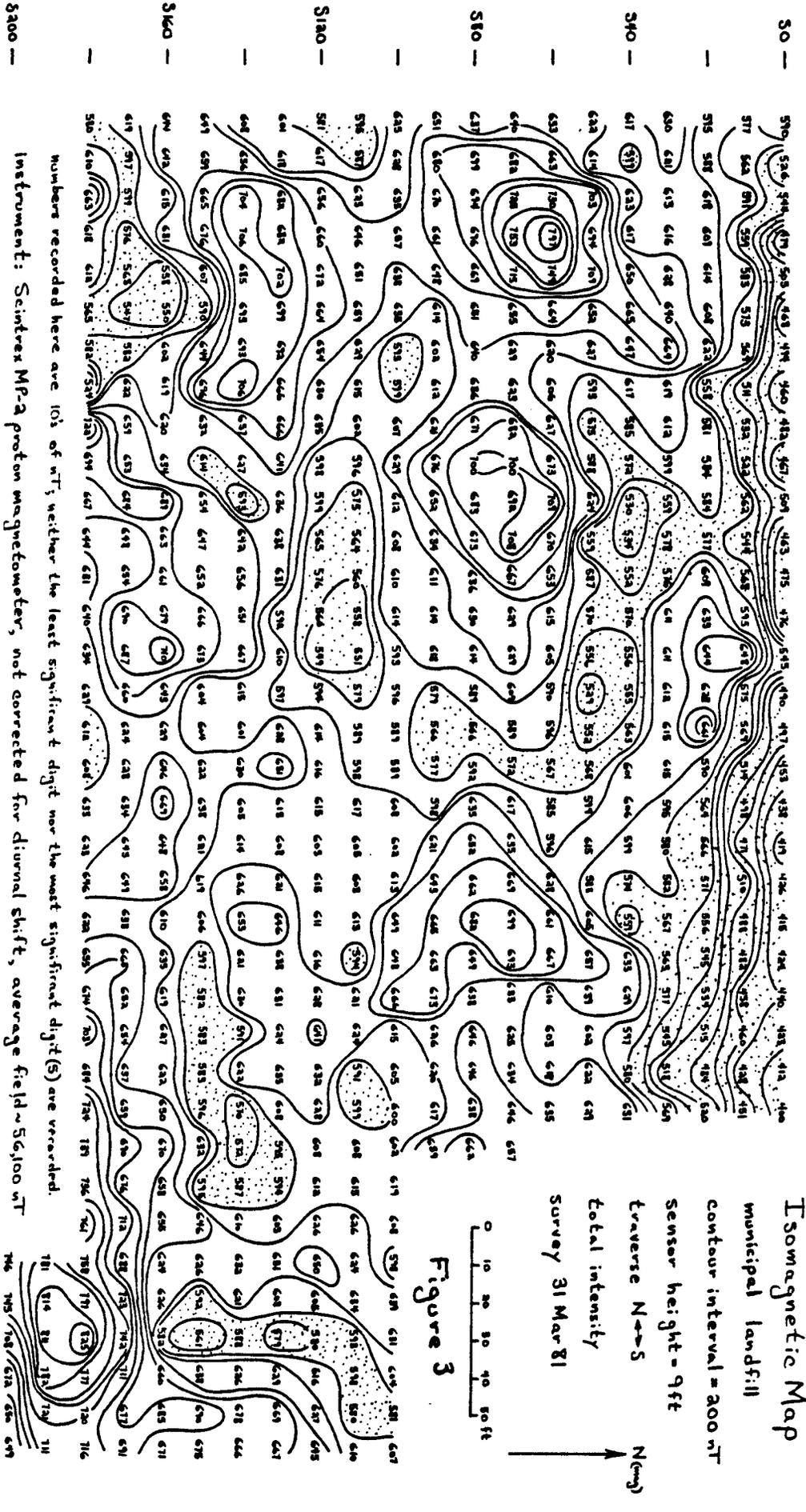


Magnetic Map, municipal landfill, survey 18 Nov 81

total flux density, B , in nanotesla (nT), contour interval = 100 nT
 sensor height = 9 ft, measurement spacing = 5 ft, no diurnal correction
 Scintrex MP-2 proton magnetometer, background field \approx 56,000 nT
 lows are stippled, traverse E \rightarrow W

T
 0 10 20

EO | E40 | E100 | E120 | E140 | E160 | E200 | E240 | E260 | E300 | E320



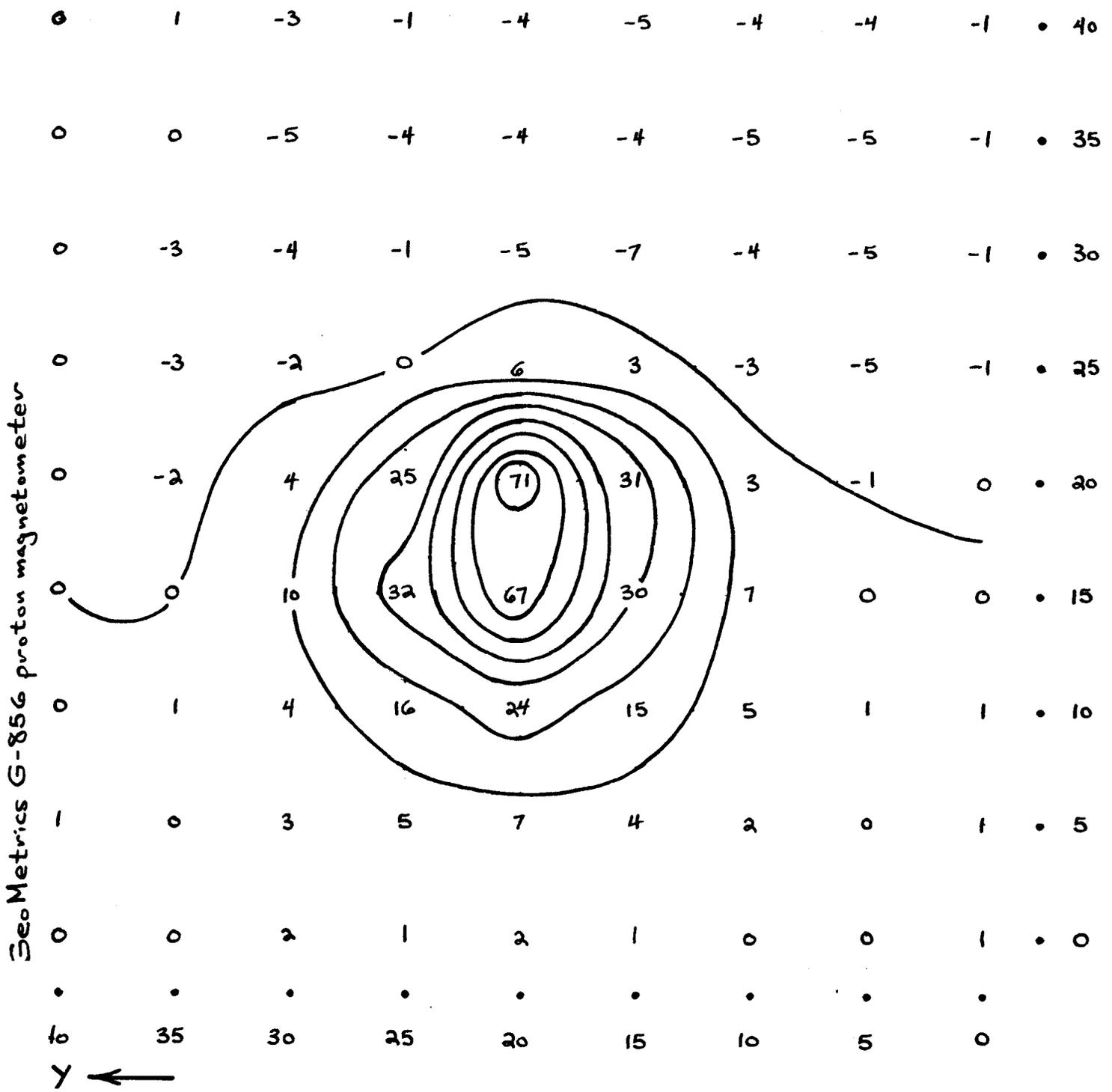
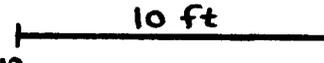
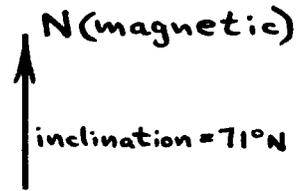
Isomagnetic Map
 municipal landfill
 contour interval = 200 nT
 sensor height = 9ft
 traverse N ← S
 total intensity
 survey 31 Mar 81

Figure 3

Numbers recorded here are 10's of nT, whether the least significant digit nor the most significant digit (5) are recorded.
 Instrument: Scintrex MFA proton magnetometer, not corrected for diurnal shift, average field ~ 56,100 nT

Magnetic Anomaly

55 gallon steel drum at $X=20, Y=20$
 sensor 10 ft above center of drum
 contour interval = 10 nT
 total flux density, field = 55,260 nT
 corrected for background & time variation

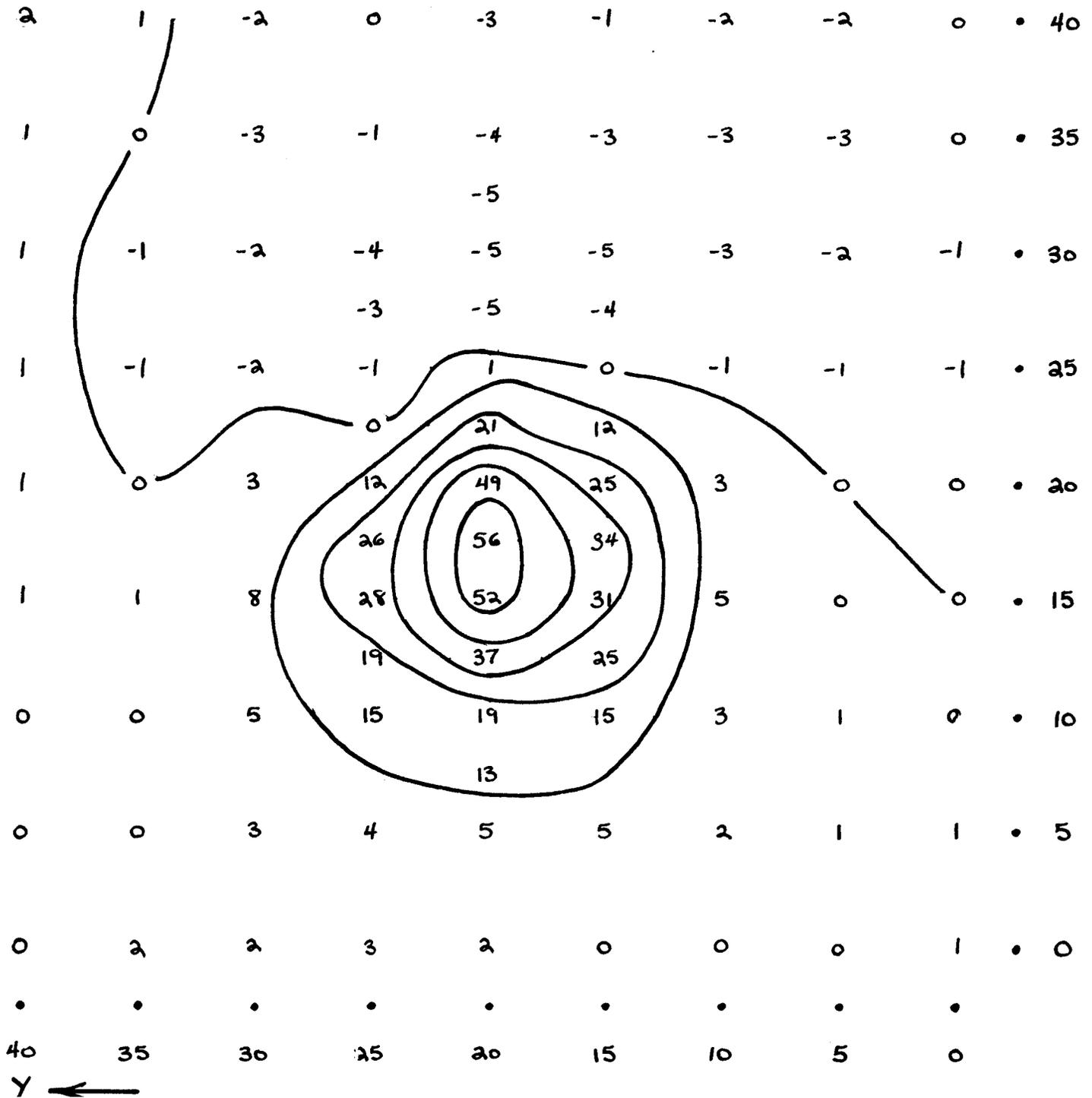
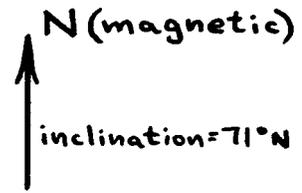


drum vertical, reference at upper north edge

Figure 4: The magnetic anomaly of a steel drum

Magnetic Anomaly

55 gallon steel drum at $X=20, Y=20$
 sensor 10 ft above center of drum
 contour interval = 10 nT
 total flux density, field = 55,260 nT
 corrected for background and
 temporal variation

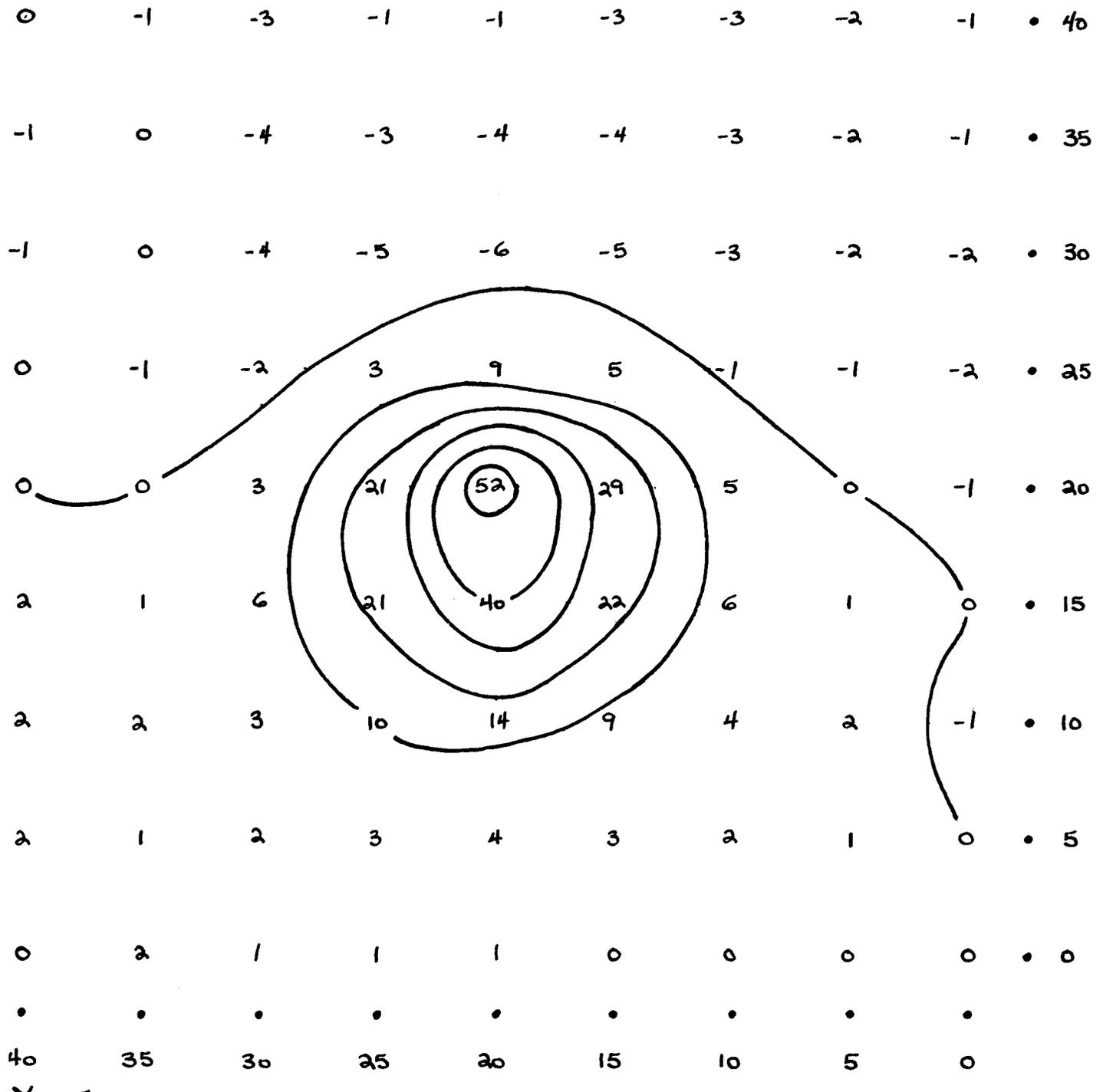
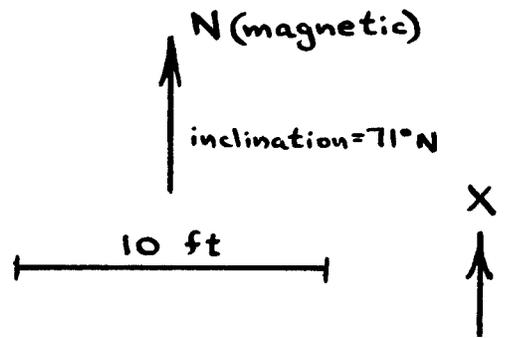


drum vertical, reference at lower south edge

Figure 5

Magnetic Anomaly

55 gallon steel drum at $X=20, Y=20$
 sensor 10 ft above center of drum
 contour interval = 10 nT
 total flux density, field = 55,260 nT
 corrected for background and
 temporal variation

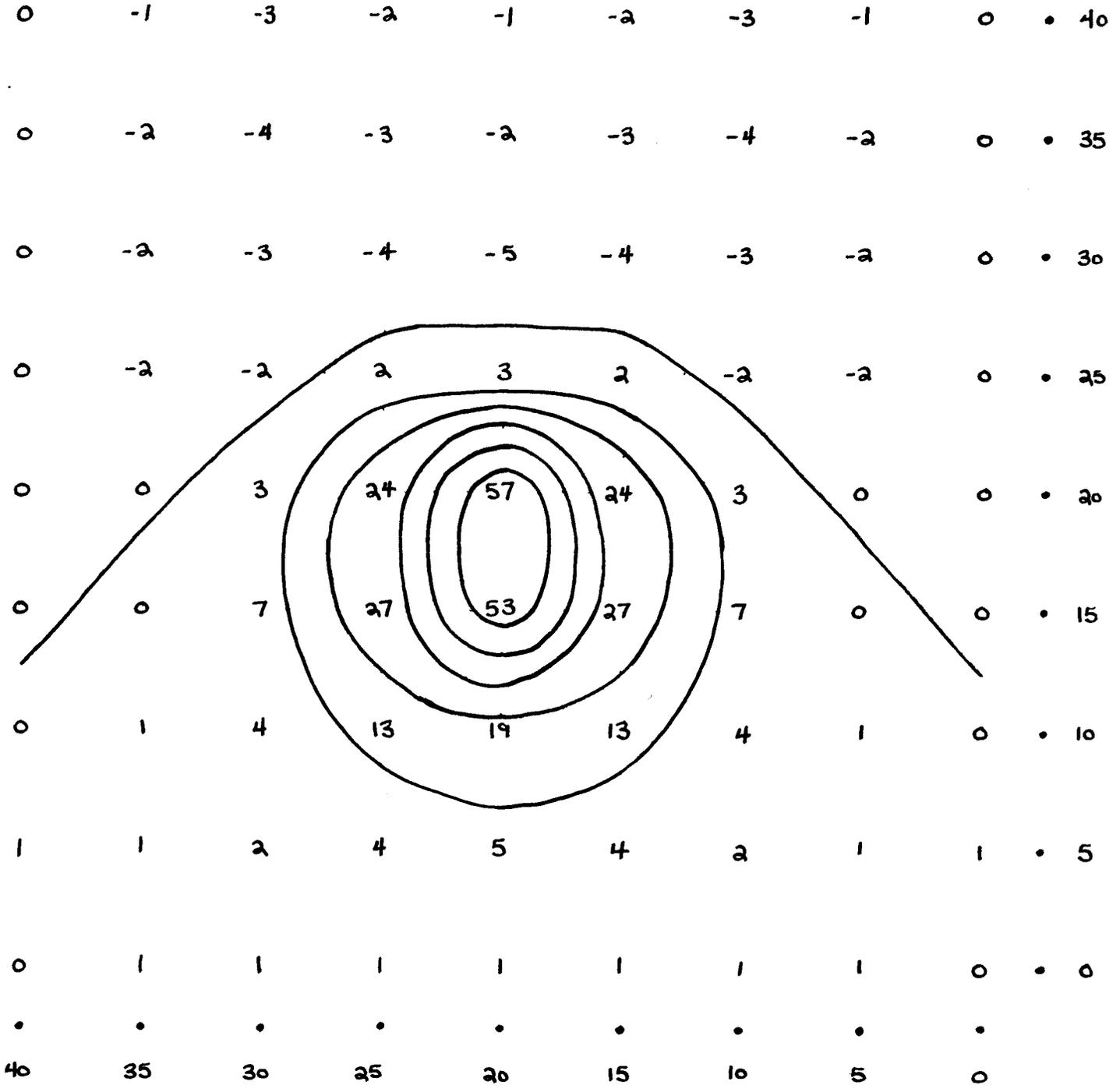
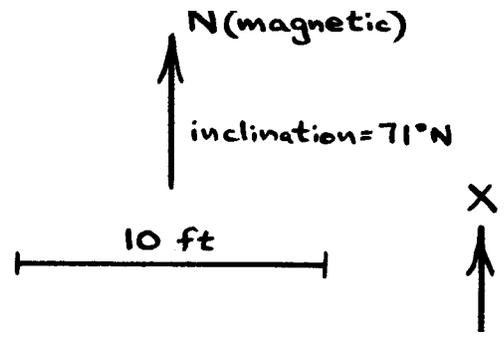


drum horizontal, reference at upper east edge

Figure 6

Magnetic Anomaly

55 gallon steel drum at $X=20, Y=20$
 sensor 10 ft above center of drum
 contour interval = 10 nT
 total flux density, field = 55,260 nT
 corrected for background and
 temporal variation

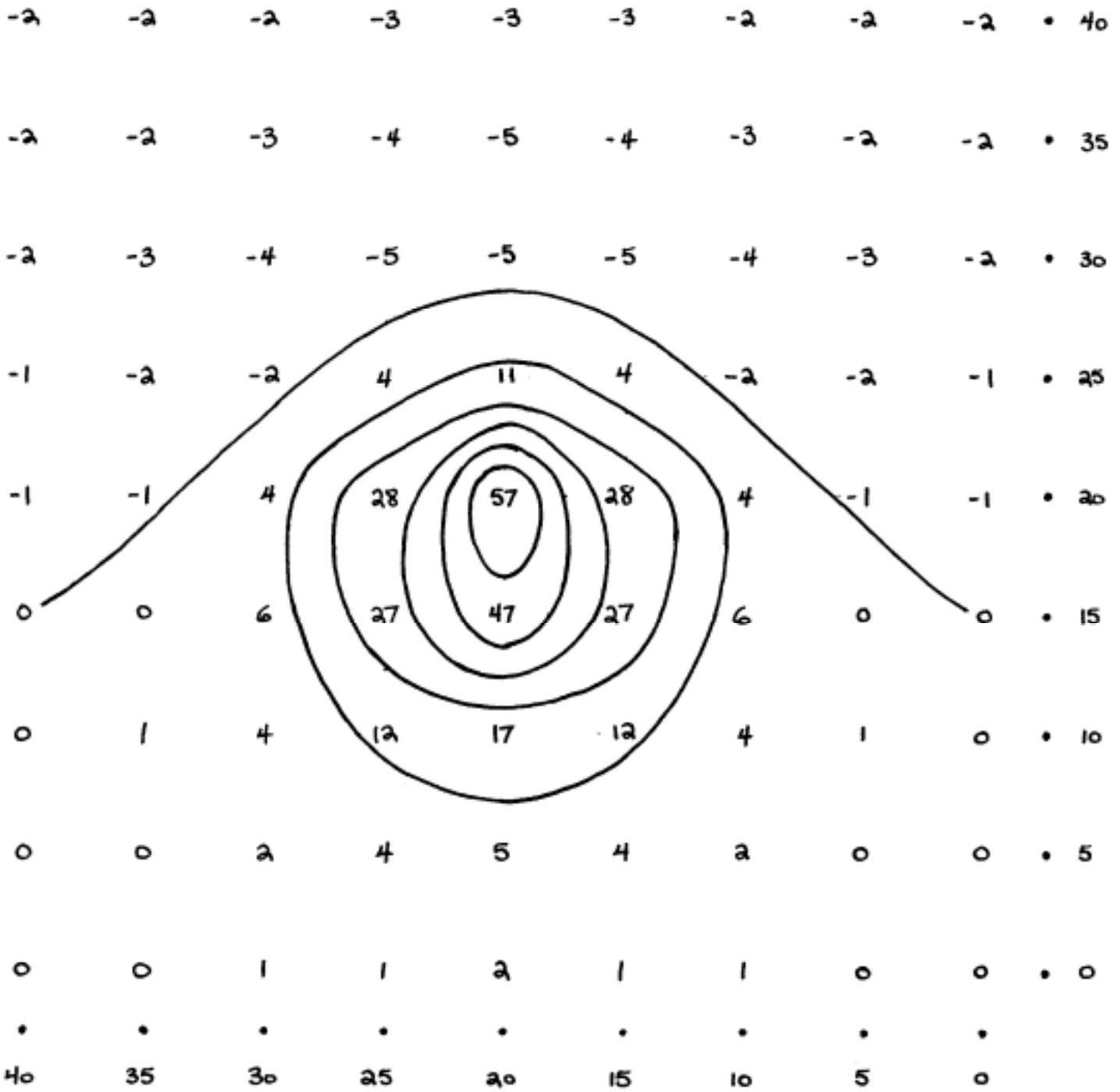
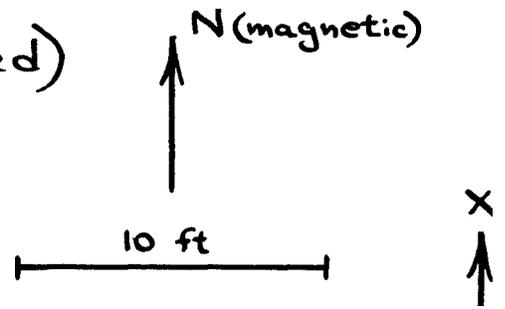


average anomaly from three orientations
 east-west anomaly averaged also

Figure 7: The average anomaly

Magnetic Anomaly (calculated)

magnetic moment = 34,000 nT·ft³
 source at X=20, Y=20 is dipole
 contour interval = 10 nT
 measurements 10 ft above source
 inclination = 71°N (earth & source)



dipole equation: $B_a = \frac{M}{r^3} [3(X \cos I - Z \sin I)^2 - r^2]$
 $r^2 = X^2 + Y^2 + Z^2$, $Z = \text{height} = 10 \text{ ft}$
 $I = \text{inclination} = 71^\circ$, $M = \text{moment} = 34,000 \text{ nT} \cdot \text{ft}^3$
 $B_a = \text{anomaly, in nT}$

Figure 8: The calculated anomaly

Analysis of a 55 gallon drum

$$V_{\text{drum}} = \text{volume of drum} = 55 \text{ gal} = 7.35 \text{ ft}^3$$

$$V_{\text{filldrum}} = \text{volume per drum in landfill} \approx 12 \text{ ft}^3$$

$$F = \text{volume fraction of earth or voids in fill} = \frac{V_{\text{filldrum}} - V_{\text{drum}}}{V_{\text{filldrum}}} = 39\%$$

$$W_{\text{drum}} = \text{weight of empty 55 gal drum} \approx 45 \text{ lb}$$

$$\rho_{\text{iron}} = \text{density of iron or steel} = 0.28 \text{ lb/in}^3 = 480 \text{ lb/ft}^3$$

$$V_{\text{iron/drum}} = \text{volume of iron in a drum} = W_{\text{drum}} / \rho_{\text{iron}} = 0.094 \text{ ft}^3$$

$$V_{\text{iron/fill}} = \text{volume fraction of iron in fill} = V_{\text{iron/drum}} / V_{\text{filldrum}} = 0.0078$$

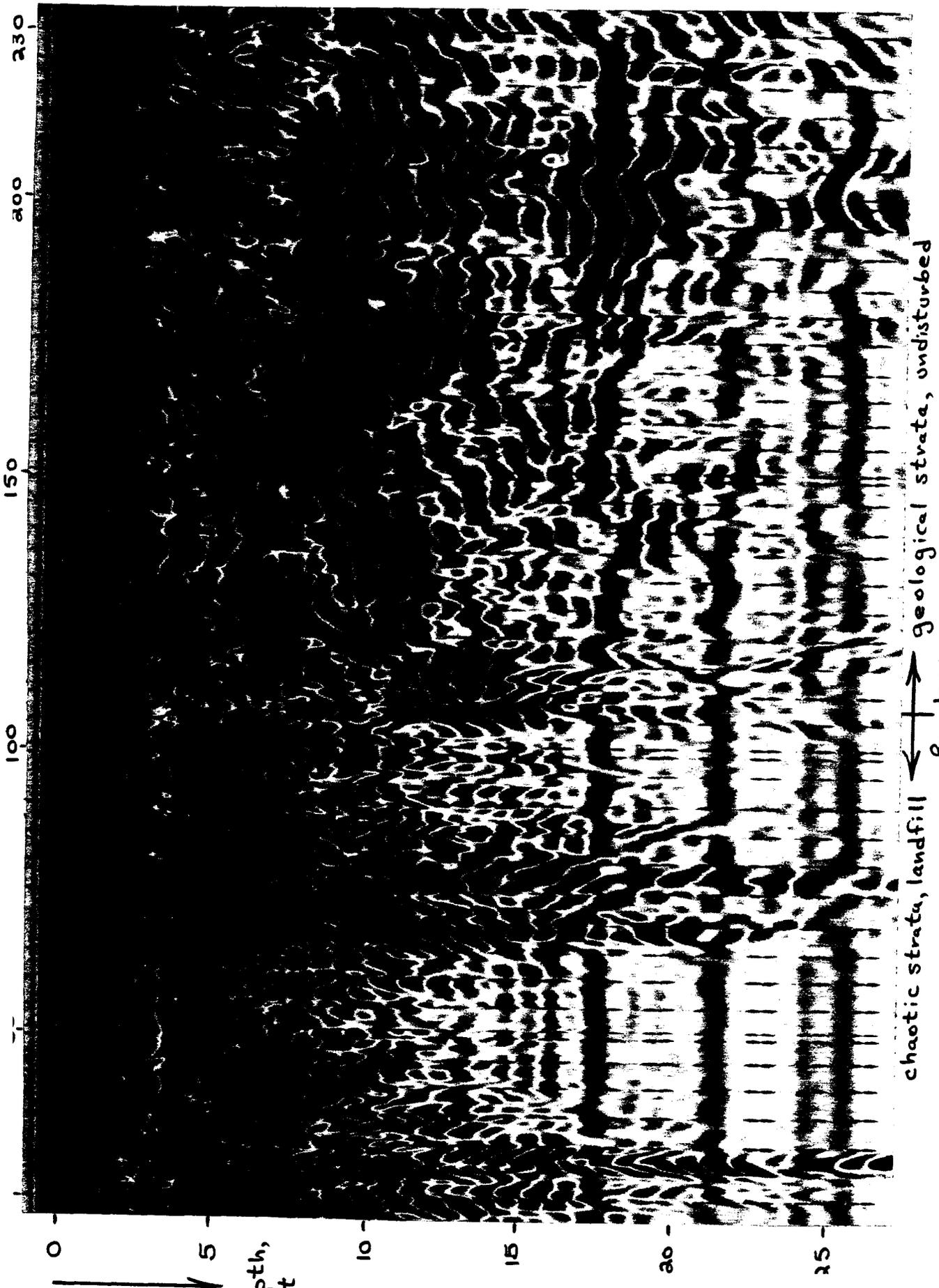
$$K_{\text{iron}} = \text{magnetic susceptibility of iron} \approx 12$$

$$\mu_r = \text{relative permeability of iron} = 1 + 4\pi K_{\text{iron}} = 152$$

$$K_{\text{eff}} = \text{effective susceptibility of fill} = K_{\text{iron}} V_{\text{iron/fill}} = 0.09$$

Figure 9

Horizontal distance, tick marks at 5 ft intervals



chaotic strata, landfill
Boundary
geological strata, undisturbed

Figure 10: A ground-penetrating radar profile of the Lipari landfill, New Jersey

scale assumes radar pulse velocity is 13.6 ns/ft

Magnetic Profiles

cylindrical models

Magnetic Anomaly
nT

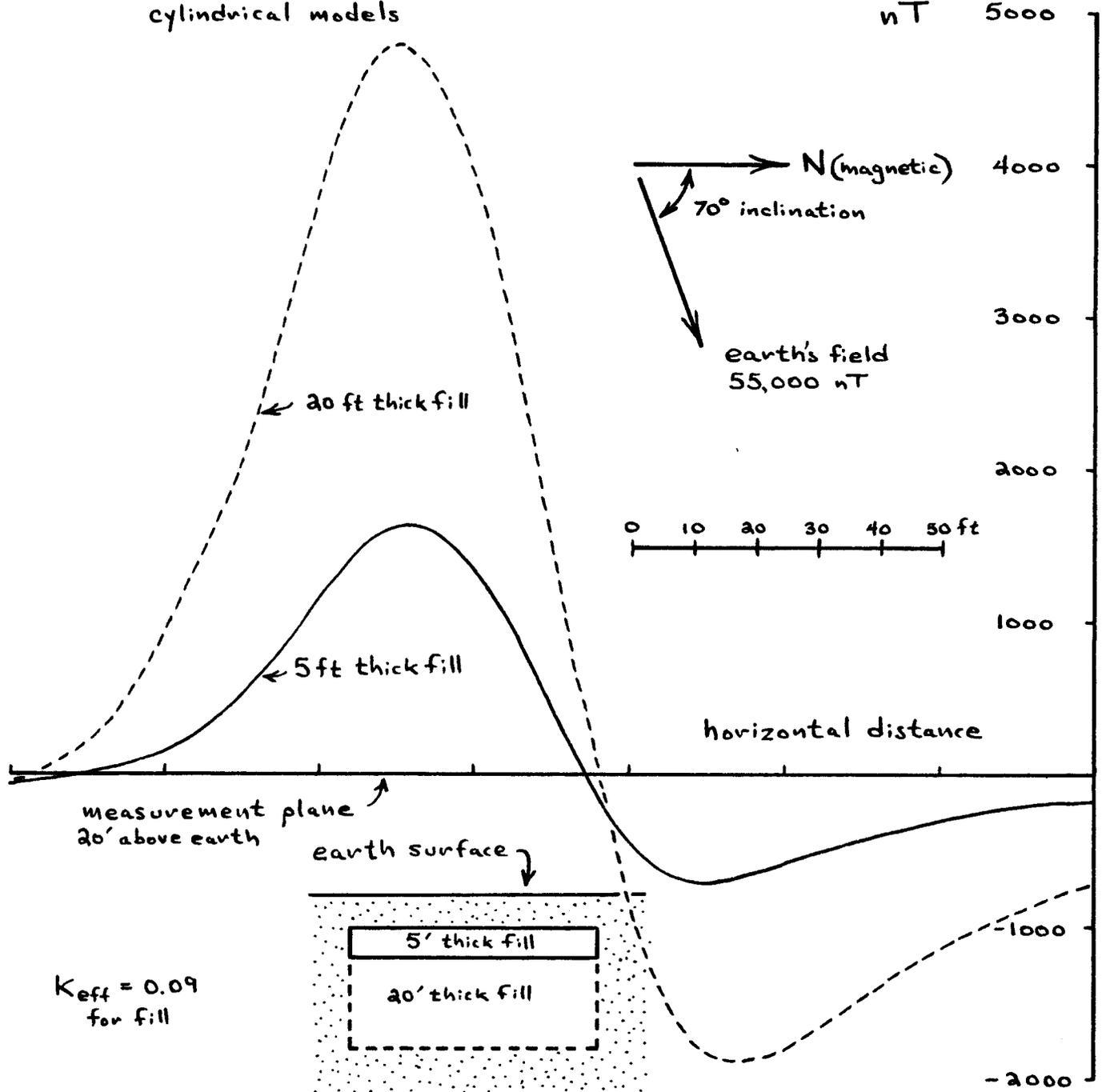


Figure 11

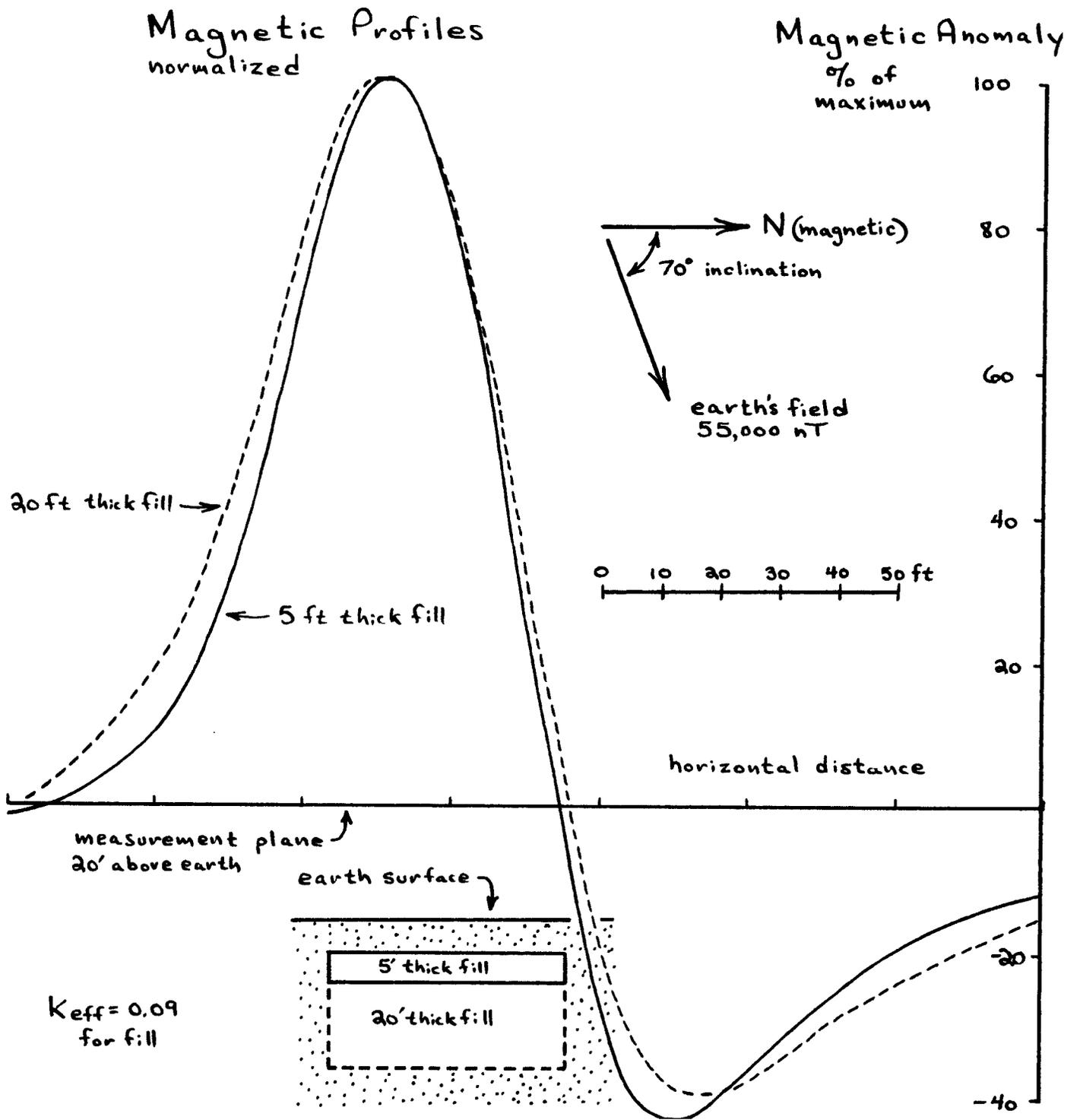


Figure 12

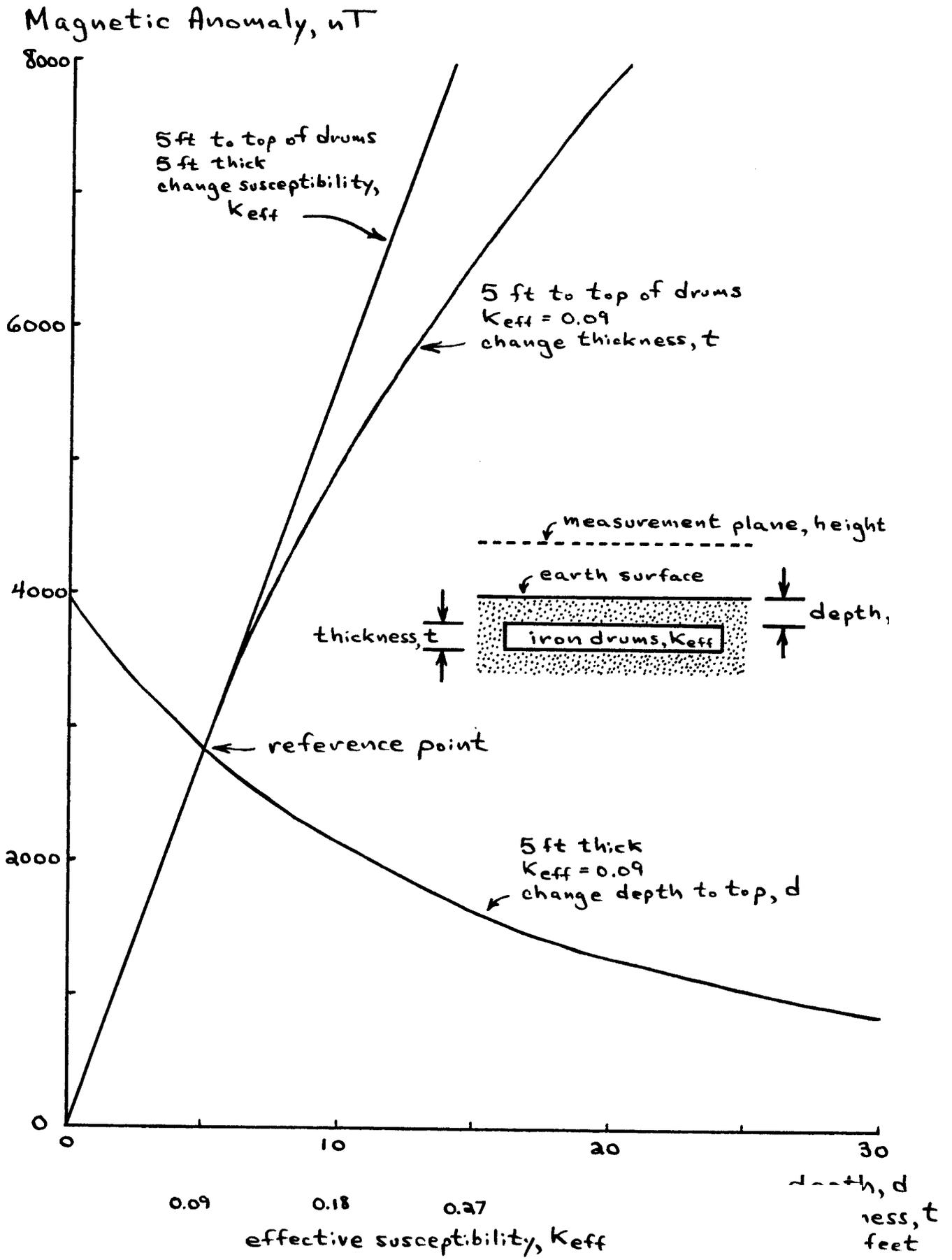


Figure 13: Factors which change the magnetic anomaly

Calculate magnetic "volume" of the fill, M_v , in ft^3

Polyhedron model

Determine effective susceptibility of drum fill, K_{eff}

Determine volume of drum fill, V , in ft^3

Calculate $M_v = K_{\text{eff}} V$

Sphere cluster model

Determine total magnetic moment of fill, M , in $\text{nT} \cdot \text{ft}^3$

Determine background magnetic field, B_e , in nT

Calculate $M_v = M/B_e$

To get the number of drums, multiply M_v by 1.6 *

To get the weight of iron, in pounds, multiply M_v by 65 **

$$\begin{array}{l} * \text{ this constant is } \frac{B_e}{M} = \frac{55,000 \text{ nT}}{34,000 \text{ nT} \cdot \text{ft}^3} \\ ** \text{ this constant is } \frac{B_e W}{M} = \frac{55,000 \text{ nT} (40 \text{ lb})}{34,000 \text{ nT} \cdot \text{ft}^3} \end{array} \left. \vphantom{\begin{array}{l} * \\ ** \end{array}} \right\} \text{ from Figure 8}$$

Figure 14